

Approximation lower bounds in L^p norm, with applications to feedforward neural networks

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We study the fundamental limits to the expressive power of neural networks. We first prove a general lower bound on the approximation error in $L^p(\mu)$ norm between two sets of functions F and G , for any $p \geq 1$ and any probability measure μ . We then instantiate this lower bound to the case where the approximation set G corresponds to a piecewise-polynomial feedforward neural network, and for several nonparametric examples of target sets F . Beside matching (known or new) upper bounds up to log factors, our lower bounds shed some light on the similarities or differences between approximation in L^p norm or in sup norm, solving an open question by DeVore et al. (2021).