ON GOODNESS-OF-FIT TESTS FOR PERTURBED DYNAMICAL SYSTEMS BASED ON A MINIMUM DISTANCE ESTIMATOR

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Goodness-of-fit test; minimum distance estimator; asymptotically distribution free tests:

We consider the problem of the construction of the goodness-of-Fit test in the case of continuous time observations of a diffusion process X^ε with small noise satisfying the equation

$$\mathrm{d}X_t = S(X_t)\,\mathrm{d}t + \varepsilon\,\mathrm{d}W_t, \qquad X_0 = x_0, \quad 0 \le t \le T,$$

where $W_t,\ 0 \le t \le T$ is a Wiener process and $S(x)$ is some unknown smooth function. Based on a minimum distance estimator (MDE), we construct an asymptotically distribution free (ADF) test in the case of the parametric null hypothesis

$$\mathcal{H}_0 : \quad S(x) = S(\vartheta, x), \qquad \vartheta \in \Theta = (a, b).$$

First, we show that the basic statistic

$$u_\varepsilon(t) = \frac{X_t - x_t(\vartheta_\varepsilon^*)}{\varepsilon\, S(\vartheta_\varepsilon^*, X_t)}, \qquad\qquad 0 \le t \le T$$

(ϑ_ε^* is the MDE) converges to some random process $u(t), 0 \le t \le T$. Then the process $u(\cdot)$ is transformed in $U(\cdot)$ which has the representation

$$U(\nu) = W(\nu) - \int_0^1 g(\vartheta, r)\,\mathrm{d}W(r) \int_0^\nu h(\vartheta, r)\,\mathrm{d}r, \quad \int_0^1 g(\vartheta, r)^2\,\mathrm{d}r = 1.$$

Further, we construct a special linear transformation $L[U](\nu) = w_\nu, 0 \le \nu \le 1$, where $w_\nu, 0 \le \nu \le 1$ is a Wiener process. Specifically, we have to solve Fredholm equation of the second kind with degenerated kernel. The solution of it gives us the desired linear transformation. This allows us to propose an ADF test $\psi_\varepsilon = 1\!\!1_{\{\Delta_\varepsilon > c_\alpha\}}$ of asymptotic ($\varepsilon \to 0$) size $\alpha \in (0,1)$ such that

$$\Delta_\varepsilon = \frac{1}{T}\int_0^T L[U_\varepsilon](t)^2\,\mathrm{d}t \Longrightarrow \int_0^1 w_\nu^2\,\mathrm{d}\nu, \qquad \mathbf{P}\left\{\int_0^1 w_\nu^2\,\mathrm{d}\nu > c_\alpha\right\} = \alpha, \quad \alpha \in (0,1)$$

Here the process $U_\varepsilon(\cdot)$ is the "empirical version" of $U(\cdot)$. We shall mention that if in our problem we use the MLE of the unknown parameter, then the limit representation is

$$U(\nu) = W(\nu) - \int_0^1 h(\vartheta, s)\,\mathrm{d}W(s) \int_0^\nu h(\vartheta, s)\,\mathrm{d}s, \qquad \int_0^1 h(\vartheta, s)^2\,\mathrm{d}s = 1$$

and our transformation coincides with the known one.

References

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