MAXIMUM LIKELIHOOD ESTIMATION FOR HESTON MODELS

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Heston model; SVJ model; maximum likelihood estimation:

Let us consider a Heston model

$$\begin{cases} \mathrm{d}Y_t = (a - bY_t)\,\mathrm{d}t + \sigma_1\sqrt{Y_t}\,\mathrm{d}W_t, \\ \mathrm{d}X_t = (\alpha - \beta Y_t)\,\mathrm{d}t + \sigma_2\sqrt{Y_t}\big(\varrho\,\mathrm{d}W_t + \sqrt{1-\varrho^2}\,\mathrm{d}B_t\big), \end{cases} \qquad t \geq 0, \tag{1}$$

where $a > 0$, $b, \alpha, \beta \in R$, $\sigma_1 > 0$, $\sigma_2 > 0$, $\varrho \in (-1, 1)$ and $(W_t, B_t)_{t\geq 0}$ is a 2-dimensional standard Wiener process. We study maximum likelihood estimator (MLE) of (a, b, α, β) based on continuous time observations $(X_t)_{t\in[0,T]}$ with $T > 0$, supposing that σ_1, σ_2 and ϱ are known. We distinguish three cases: subcritical (also called ergodic), critical and supercritical according to $b > 0$, $b = 0$ and $b < 0$. In the subcritical case, asymptotic normality is proved for all the parameters, while in the critical and supercritical cases, non-standard asymptotic behavior is described. Our results can be found in [1].

Next, let us consider a jump-type Heston model (also called a stochastic volatility with jumps model, SVJ model)

$$\begin{cases} \mathrm{d}Y_t = \kappa(\theta - Y_t)\,\mathrm{d}t + \sigma_1\sqrt{Y_t}\,\mathrm{d}W_t, \\ \mathrm{d}S_t = \mu S_t\,\mathrm{d}t + S_t\sqrt{Y_t}\big(\varrho\,\mathrm{d}W_t + \sqrt{1-\varrho^2}\,\mathrm{d}B_t\big) + S_{t-}\,\mathrm{d}L_t, \end{cases} \qquad t \geq 0, \tag{2}$$

where $\kappa, \theta > 0$, $(L_t)_{t\geq 0}$ is a purely non-Gaussian Lévy process independent of $(W_t, B_t)_{t\geq 0}$ with Lévy–Khintchine representation

$$\mathrm{E}(\mathrm{e}^{\mathrm{i}uL_1}) = \exp\left\{\mathrm{i}\gamma u + \int_{-1}^{\infty} (\mathrm{e}^{\mathrm{i}uz} - 1 - \mathrm{i}uz\mathbb{1}_{[-1,1]}(z))\,m(\mathrm{d}z)\right\}, \qquad u \in R,$$

where $\gamma \in R$ and m is a Lévy measure concentrating on $(-1, \infty)$ with $m(\{0\}) = 0$. We point out that the first coordinate process in (1) can be subcritical, critical or supercritical, but in (2) it can only be subcritical (since $\kappa > 0$). Moreover, the second coordinate process in (2) is the price process, while in (1) it is the log-price process. We study MLE of (θ, κ, μ) based on continuous time observations $(S_t, L_t)_{t\in[0,T]}$ with $T > 0$, supposing that σ_1, ϱ, γ and the Lévy measure m are known. We prove strong consistency and asymptotic normality for all admissible parameter values except $\theta\kappa = \sigma_1^2/2$, where we show only weak consistency and non-normal asymptotic behavior. Our results can be found in [2].

References

[1] Barczy, M., Pap, G. (2016) *Asymptotic properties of maximum-likelihood estimators for Heston models based on continuous time observations*, Statistics 50(2), 389–417.

[2] Barczy, M., Ben Alaya, M., Kebaier, A., Pap, G. (2015) *Asymptotic behavior of maximum likelihood estimators for a jump-type Heston model*, Arxiv: 1509.08869