

# STATISTICAL INFERENCE OF 2-TYPE GALTON–WATSON PROCESSES WITH IMMIGRATION

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*Galton–Watson branching process with immigration; conditional least squares estimator:*

In this talk the asymptotic behavior of the conditional least squares estimators of the offspring mean matrix for a 2-type positively regular Galton–Watson branching process with immigration is described. We also study this question for a natural estimator of the spectral radius of the offspring mean matrix, which we call criticality parameter.

Let  $(X_k)_{k \in \{0,1,\dots\}}$  be a 2-type (2-dimensional) Galton–Watson branching process with immigration and with initial value  $X_0 = 0$  given by

$$X_k = \sum_{j=1}^{X_{k-1,1}} \xi_{k,j,1} + \sum_{j=1}^{X_{k-1,2}} \xi_{k,j,2} + \varepsilon_k, \quad k \in \{1, 2, \dots\}.$$

It is referred to respectively as *subcritical*, *critical* or *supercritical* if  $\rho < 1$ ,  $\rho = 1$  or  $\rho > 1$ , where  $\rho$  denotes the spectral radius of the offspring mean matrix  $m_\xi$ . Assume that the immigration mean vector  $m_\varepsilon$  is known. The conditional least squares estimator  $\widehat{m}_\xi^{(n)}$  of  $m_\xi$  based on a sample  $X_1, \dots, X_n$  can be obtained by minimizing the sum of squares

$$\sum_{k=1}^n \|X_k - E(X_k | \mathcal{F}_{k-1})\|^2 = \sum_{k=1}^n \|X_k - m_\xi X_{k-1} - m_\varepsilon\|^2$$

with respect to  $m_\xi$  over  $R^{2 \times 2}$ . In the critical case we obtain the asymptotic distribution of  $\widehat{m}_\xi^{(n)}$  with rate  $\sqrt{n}$  to a limiting distribution represented as a functional of the Feller diffusion given by

$$d\mathcal{Y}_t = \langle u, m_\varepsilon \rangle dt + \sqrt{\langle V_\xi u, u \rangle} \mathcal{Y}_t^+ d\mathcal{W}_t, \quad t \in [0, \infty), \quad \mathcal{Y}_0 = 0,$$

where  $u$  is a left Perron eigenvector of  $m_\xi$  corresponding to the eigenvalue 1, and  $V_\xi$  is a mixed offspring variance matrix. In the subcritical case  $\widehat{m}_\xi^{(n)}$  is asymptotically normal, while in the supercritical case it is asymptotically mixed normal.

## References

- [1] Körmendi, K., Pap, G. (2015) *Statistical inference of 2-type critical Galton–Watson processes with immigration*, Available on the ArXiv: <http://arxiv.org/abs/1502.04900>.