Introduction to random fields and scale invariance: Lecture IV

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Outlines

1. Random fields and scale invariance
2. Sample paths properties
3. Simulation and estimation
4. Geometric construction and applications

Lecture 4 :

1. Geometric construction
   1. Random measures
   2. Chentsov's representation : Lévy and Takenaka constructions
   3. Fractional Poisson field
2. Application in medical imaging analysis
   1. Osteoporosis and bone radiographs
   2. Mammograms and density analysis

Random Measures

Let $(\Omega, \mathcal{A}, \mathbb{P})$ be a probability space. Let μ be a σ-finite nonnegative measure on $(\mathbb{R}^k, \mathcal{B}(\mathbb{R}^k))$ with $k \geq 1$ and set

$$\mathcal{E}_\mu = \{A \in \mathcal{B}(\mathbb{R}^k) \text{ s.t. } \mu(A) < +\infty\}.$$

A random measure M is a stochastic process $M = \{M(A); A \in \mathcal{E}_\mu\}$ satisfying

- For all $A \in \mathcal{E}_\mu$, $M(A)$ is a real random variable ;
- For $A_1, \ldots, A_n \in \mathcal{E}_\mu$ disjoint sets the r.v. $M(A_1), \ldots, M(A_n)$ are independant ;
- For $(A_n)_{n \in \mathbb{N}}$ disjoint sets s.t. $\underset{n \in \mathbb{N}}{\cup} A_n \in \mathcal{E}_\mu$,

$$M(\underset{n \in \mathbb{N}}{\cup} A_n) = \sum_{n \in \mathbb{N}} M(A_n) \text{ a.s.}$$

Poisson random measures

A Poisson random mesure with intensty μ is a r.m. N such that

$$N(A) \sim \mathcal{P}(\mu(A)).$$

In this case

$$N = \sum_{i \in I} \delta_{T_i},$$

where $\Phi = (T_i)_{i \in I}$ is a countable family of random variables with values in $\mathbb{R}^k$ called Poisson point process on $\mathbb{R}^k$ with intensity μ.

Exple : $k = 1$, $\mu = \lambda$Lebesgue, $(N([0, t]))_{t \geq 0}$ Poisson process of intensity λ and Φ corresponds to the jumps of the Poisson process.

Gaussian random measures

A Gaussian random measure with intensity μ is a r.m. W such that

$$W(A) \sim \mathcal{N}(0, \mu(A)).$$

In this case, for all $A, B \in \mathcal{E}_\mu$,

$$\begin{aligned} \mathrm{Cov}\left(W(A), W(B)\right) &= \mu(A \cap B) \\ &= \frac{1}{2}\left(\mu(A) + \mu(B) - \mu(A \Delta B)\right) \end{aligned}$$

Rk : true as soon as M is of second order st $\mathrm{Var}(M(A)) = \mu(A)$ and so for N Poisson r.m. of intensity μ.

Exple : $k = 1$, $\mu = \lambda$Lebesgue, $B = (W([0, t]))_{t \geq 0}$ is a Brownian motion with diffusion λ.
Conversely one can define $W(A) = \int_0^{+\infty} \mathbf{1}_A(t) dB_t$.

Central limit theorem in high intensity

If $N^{(1)}, \dots, N^{(n)}$ are independant Poisson r.m. with the same intenity μ

$$\sum_{i=1}^{n} N^{(i)} \text{ is a Poisson random measure with intensity } n\mu.$$

By CLT, we deduce that if N_λ is a Poisson r.m. with intensity $\lambda\mu$ and W is a Gaussian r.m. with intensity μ,

$$\left(\lambda^{-1/2}\left(N_\lambda(A) - \lambda\mu(A)\right)\right)_{A\in\mathcal{E}_\mu} \xrightarrow[\lambda\to+\infty]{fdd} (W(A))_{A\in\mathcal{E}_\mu}\,.$$

Donsker invariance principle

Let $(X_j)_{j\in\mathbb{Z}^k}$ be an iid sequence $\mathbb{E}(X_j) = 0$ and $\mathrm{Var}(X_j) = 1$.
Let $\mathcal{A} \subset \mathcal{B}(\mathbb{R}^k)$ and define the set-indexed process for $A \in \mathcal{A}$,

$$S(A) = \sum_{j\in A} X_j = \sum_{j\in\mathbb{Z}^k} X_j \delta_j(A).$$

By CLT, for W a Gaussian r.m. with intensity Leb, we get

$$\left(n^{-k/2} S(nA)\right)_{A\in\mathcal{A}} \xrightarrow[n\to+\infty]{fdd} (W(A))_{A\in\mathcal{A}},$$

Alexander and Pyke [86] obtained invariance principle considering the smoothed version

$$S(A) = \sum_{j\in\mathbb{Z}^k} \mathrm{Leb}(A \cap R_j) X_j \text{ with } R_j = \prod_{i=1}^{k} [j_i, j_i + 1].$$

and $\mathcal{A} \subset \{B \in \mathcal{B}(\mathbb{R}^k); \mathrm{Leb}(\partial B) = 0\}$.

Self-similar measures

Let μ be a σ-finite nonnegative measure on $(\mathbb{R}^k, \mathcal{B}(\mathbb{R}^k))$.
Ass. 1 : $\exists \beta > 0$ s.t. $\forall A$ with $\mu(A) < +\infty, n \in \mathbb{N}^*$, $\mu(nA) = n^\beta \mu(A)$
We define

$$S(A) = \sum_{j \in \mathbb{Z}^k} \mu(A \cap R_j)^{1/2} X_j \text{ with } R_j = \prod_{i=1}^{k} [j_i, j_i + 1].$$

Assuming $X_j \sim \mathcal{N}(0,1)$, $n^{-\beta/2} S(nA) \sim \mathcal{N}(0, \mu(A))$, since by Ass. 1,

$$\mu(A) = n^{-\beta} \sum_{j \in \mathbb{Z}^k} \mu(nA \cap R_j) \text{ with } R_j = \prod_{i=1}^{k} [j_i, j_i + 1].$$

Ass 2 : $\mu << \text{Leb}$ and $\mathcal{A} \subset \{A \in \mathcal{B}(\mathbb{R}^k); \mu(A) < \infty \text{ and } Leb(\partial A) = 0\}$
Then, for W a Gaussian r.m. with intensity μ

$$\left(n^{-\beta/2} S(nA)\right)_{A \in \mathcal{A}} \xrightarrow[n \to +\infty]{fdd} (W(A))_{A \in \mathcal{A}}.$$

Extension to the iid case- Lindeberg's type condition

Ass 3 : Let $\pi(j) = \min_{1 \le i \le k}(|j_i|)$, we assume that

(a) $\displaystyle \limsup_{\pi(j) \to +\infty} \mu(R_j) < +\infty$;

(b) $\forall e \in \mathbb{Z}^k$ with $|e| = 1$, $\mu(R_{j+e}) = \mu(R_j) + \underset{\pi(j) \to +\infty}{o}(\mu(R_j))$,

Under Ass 1-3, Lindeberg's type condition (CLT without id) is satisfied, for $(X_j)_{j \in \mathbb{Z}^k}$ iid with $\mathbb{E}(X_j) = 0$ and $\text{Var}(X_j) = 1$,

$$\left(n^{-\beta/2} S(nA)\right)_{A \in \mathcal{A}} \xrightarrow[n \to +\infty]{fdd} (W(A))_{A \in \mathcal{A}} .$$

Extension under a weak dependence assumption (2-stability Wu [05]) to the case where $(X_j)_{j \in \mathbb{Z}^k}$ is stationary sequence.
[HB, **O. Durieu**, *Trans. AMS* (2014)]

Chentsov's type representation [Samorodnitsky, Taqqu, (1994)]

Let M be a r.m. associated with μ on $\mathbb{R}^k$ and $\mathcal{V} = \{V_x; x \in \mathbb{R}^d\}$ for $d \geq 1$, with $\mu(V_x) < \infty$. The random field

$$X_x = M(V_x),\ x \in \mathbb{R}^d$$

is called Chentsov random field associated with M and $\mathcal{V}$. If M is of second order s.t. $\mathrm{Var}(M(A)) = \mu(A)$ then

$$\mathrm{Var}(X_x - X_y) = \mu(V_x \Delta V_y).$$

1. X has stationary increments $\Rightarrow \mu(V_x \Delta V_y) = \mu(V_{x-y} \Delta V_0)$;
2. X is isotropic $\Rightarrow \mu(V_{Rx}) = \mu(V_x)$, $\forall$ vectorial rotation R;
3. X is H-self-similar $\Rightarrow \mu(V_{cx}) = c^{2H}\mu(V_x)$, $\forall c > 0$.

Chentsov's type representation

If X is H self-similar with stationary increments then $H \in [0, 1/2]$. If moreover X is isotropic

$$\Rightarrow \mu(V_x \Delta V_y) = C\|x-y\|^{2H},\ \ t,s \in \mathbb{R}^d$$

Lévy Chentsov's construction (1948 & 1957) for $H = 1/2$:

- $\forall x \in \mathbb{R}^d, V_x = B\left(\frac{x}{2}, \frac{\|x\|}{2}\right) = \left\{z \in \mathbb{R}^d\ :\ \|z - \frac{x}{2}\| < \frac{\|x\|}{2}\right\}$.
- $\mu(dz) = \|z\|^{-d+1}dz$, $2H = 1$-self-similar on $\mathbb{R}^d$.

In polar coordinates $V_x = \left\{(r,\theta) \in \mathbb{R}_+ \times S^{d-1}\ :\ 0 < r < \theta \cdot x\right\}$

Lévy Chentsov's construction for $H = 1/2$

Then,

$$\mu(V_x) = \int_{S^{d-1}} \int_{\mathbb{R}_+} 1_{\{r<\theta\cdot x\}} dr d\theta = \frac{1}{2} \int_{S^{d-1}} |\theta \cdot x| d\theta = \frac{c_d}{2} \|x\|.$$

Moreover,

$$\begin{aligned} \mu(V_x \cap V_y^c) &= \int_{S^{d-1}} \int_{\mathbb{R}_+} 1_{\{\theta\cdot y \le r < \theta\cdot x\}} dr d\theta \\ &= \int_{0<\theta\cdot y<\theta\cdot x} \theta \cdot (x-y) d\theta + \int_{\theta\cdot y<0<\theta\cdot x} \theta \cdot x d\theta. \end{aligned}$$

Similarly, by change of variable,

$$\mu(V_y \cap V_x^c) = \int_{\theta\cdot y<\theta\cdot x<0} |\theta \cdot (x-y)| d\theta + \int_{\theta\cdot y<0<\theta\cdot x} (-\theta \cdot y)\, d\theta,$$

so that

$$\mu(V_x \Delta V_y) = \frac{1}{2} \int_{S^{d-1}} |\theta \cdot (x-y)| d\theta = \frac{c_d}{2} \|x-y\|.$$

Takenaka's construction (1987) for $H \in (0, 1/2)$

- $\forall x \in \mathbb{R}^d, \mathcal{C}_x = \left\{(z,r) \in \mathbb{R}^d \times \mathbb{R} : \ \|z - x\| \leq r\right\}$ and $V_x = \mathcal{C}_x \Delta \mathcal{C}_0$.
- $\mu_H(dz, dr) = r^{2H-d-1}\mathbf{1}_{r>0} dzdr$ $2H$-self-similar on $\mathbb{R}^d \times \mathbb{R}$.

$$\begin{aligned} \mu_H(\mathcal{C}_x \cap \mathcal{C}_0^c) &= \frac{1}{d-2H}\int_{\|z-x\|<\|z\|} \left(\|z-x\|^{2H-d} - \|z\|^{2H-d}\right) dz \\ &= C_{H,d}\|x\|^{2H} = \mu_H(\mathcal{C}_0 \cap \mathcal{C}_x^c). \end{aligned}$$

Rk : $V_x \Delta V_y = \mathcal{C}_x \Delta \mathcal{C}_y$, $\mu_H(\mathcal{C}_x \Delta \mathcal{C}_y) = \mu_H(\mathcal{C}_{x-y} \Delta \mathcal{C}_0)$ but $\mu_H(\mathcal{C}_x) = +\infty$.

Invariance principle

If $(X_j)_{j\in\mathbb{Z}^{d+1}}$ is a centered stationary sequence 2-stable, then

$$\left(n^{-H}\sum_{j\in\mathbb{Z}^{d+1}}\mu_H(nV_x\cap R_j)^{1/2}X_j\right)_{x\in\mathbb{R}^d}\xrightarrow[n\to+\infty]{fdd}(\sigma W_H(V_x))_{x\in\mathbb{R}^d}\,,$$

with

- $\sigma^2=\sum_{j\in\mathbb{Z}^{d+1}}\mathrm{Cov}(X_0,X_j)$
- W_H is a Gaussian r.m. on $\mathbb{R}^d\times\mathbb{R}$ of intensity μ_H
- $(W_H(V_x))_{t\in\mathbb{R}^d}=\left(\sqrt{C_{H,d}}B_H(x)\right)_{x\in\mathbb{R}^d}$

where B_H is the Levy Fractional Brownian field characterized by

$$\mathrm{Cov}(B_H(x),B_H(y))=\frac{1}{2}\left(\|x\|^{2H}+\|y\|^{2H}-\|x-y\|^{2H}\right),$$

Poisson case

When $N_{\lambda,H}$ is a Poisson r.m. on $\mathbb{R}^d \times \mathbb{R}$ with intensity $\lambda\mu_H$ for $\lambda > 0$,

$$N_{\lambda,H}(\mathcal{C}_x \Delta \mathcal{C}_0) = N_{\lambda,H}(\mathcal{C}_x \cap \mathcal{C}_0^c) + N_{\lambda,H}(\mathcal{C}_x^c \cap \mathcal{C}_0)$$

We define the centered fractional Poisson field on $\mathbb{R}^d$ by :

$$\begin{aligned} F_{\lambda,H}(x) &= N_{\lambda,H}(\mathcal{C}_x \cap \mathcal{C}_0^c) - N_{\lambda,H}(\mathcal{C}_x^c \cap \mathcal{C}_0) \\ &= \int_{\mathbb{R}^d \times \mathbb{R}} \left(\mathbf{1}_{B(z,r)}(x) - \mathbf{1}_{B(z,r)}(0)\right) N_{\lambda,H}(dz, dr). \end{aligned}$$

Then $(F_{\lambda,H}(x))_{x\in\mathbb{R}^d}$ is centered, with stationary increments, isotropic with covariance

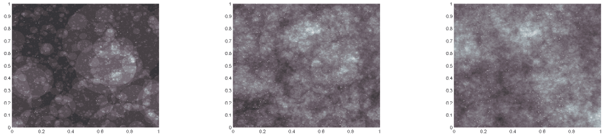
$$\mathrm{Cov}(F_{\lambda,H}(x), F_{\lambda,H}(y)) = \frac{\lambda C_{H,d}}{2}\left(\|x\|^{2H} + \|y\|^{2H} - \|x-y\|^{2H}\right).$$

This field is not self-similar but

$$(F_{\lambda,H}(cx))_{x\in\mathbb{R}^d} \overset{fdd}{=} (F_{\lambda c^{2H},H}(x))_{x\in\mathbb{R}^d},\ \forall c > 0.$$

Properties

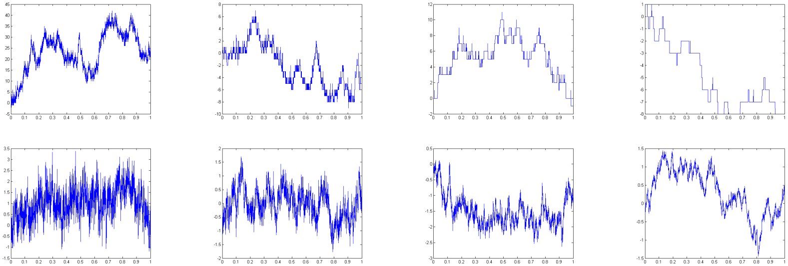
- Finite-dimensional distributions of $(F_{\lambda,H}(x))_{x\in\mathbb{R}^d}$ are characterized by $(d+1)$-dimensional ones [Sato,1991].
- CLT $(\lambda^{-1/2}F_{\lambda,H}(x))_{x\in\mathbb{R}^d} \underset{\lambda\to+\infty}{\overset{fdd}{\longrightarrow}} (\sqrt{C_{H,d}}B_H(x))_{x\in\mathbb{R}^d}$



- For H_k vector subspace of dimension $k\leq d$
$(F_{\lambda,H}(x_0+t)-F_{\lambda,H}(x_0))_{t\in H_k} \overset{fdd}{=} (F^k_{C_{H,d}C^{-1}_{H,k}\lambda,H}(t))_{t\in\mathbb{R}^k}$, with F^k a fractional Poisson field defined on $\mathbb{R}^k$.

The case of dimension 1

Sample paths Poisson (top) vs Gaussian (bottom)



$H = 0.1$ $H = 0.2$ $H = 0.3$ $H = 0.4$

Quadratic Variations

For $u \in \mathbb{N}^*$, quadratic variations of $F_{\lambda,H}$ with step u :

$$V^F_{\lambda,n}(u) = \frac{1}{n}\sum_{k=0}^{n-1}\left(F_{\lambda,H}(k+u) - F_{\lambda,H}(k)\right)^2,$$

and $V^B_{\lambda,n}(u)$ quadratic variations of $B_{\lambda,H}$ with step u for $B_{\lambda,H}$ a fBm with same covariance as $F_{\lambda,H}$.

- $\mathbb{E}(V^F_{\lambda,n}(u)) = \mathrm{Var}(F_{\lambda,H}(u)) = \lambda\, C_{H,1}u^{2H} = \mathbb{E}(V^B_{\lambda,n}(u))$;
- [HB, Demichel, Estrade, ECP 2013] $\exists$ $v_{1,u}(H) > 0$ and $v_{2,u}(H) > 0$ tq

$$\mathrm{Var}\left(V^F_{\lambda,n}(u)\right) \underset{n\to+\infty}{\sim} \left(\lambda v_{1,u}(H) + 2\lambda^2 v_{2,u}(H)\right) n^{-1}$$

- [Breuer, Major, 1983]

$$\mathrm{Var}\left(V^B_{\lambda,n}(u)\right) \underset{n\to+\infty}{\sim} 2\lambda^2 v_{2,u}(H) n^{-1},$$

$$\text{and } \sqrt{n}\left(V^B_{\lambda,n}(u) - \mathbb{E}(V^B_{\lambda,n}(u))\right) \xrightarrow[n\to+\infty]{d} \mathcal{N}(0, 2\lambda^2 v_{2,u}(H)).$$

Estimation on a fixed interval

For $u \in \mathbb{N}^*$, we replace $V^F_{\lambda,n}(u)$ by :

$$W^F_{\lambda,n}(u) = \frac{1}{n}\sum_{k=0}^{n-1}\left(F_{\lambda,H}\left(\frac{k+u}{n}\right) - F_{\lambda,H}\left(\frac{k}{n}\right)\right)^2,$$

Then $\mathbb{E}(W^F_{\lambda,n}(u)) = n^{-2H}\mathbb{E}(V^F_{\lambda,n}(u)) = \lambda\, C_{H,1}\, u^{2H} n^{-2H} = \mathbb{E}(W^B_{\lambda,n}(u))$.

- $W^F_{\lambda,n}(u) \stackrel{d}{=} V^F_{\lambda\, n^{-2H},n}(u)$ and

$$\operatorname{Var}\left(\frac{W^F_{\lambda,n}(u)}{\mathbb{E}(W^F_{\lambda,n}(u))}\right) \underset{n\to+\infty}{\sim} \frac{v_{1,u}(H)}{\lambda\, C^2_{H,1}\, u^{4H}}\, n^{-(1-2H)}.$$

- $W^B_{\lambda,n}(u) \stackrel{d}{=} n^{-2H} V^B_{\lambda,n}(u)$ and

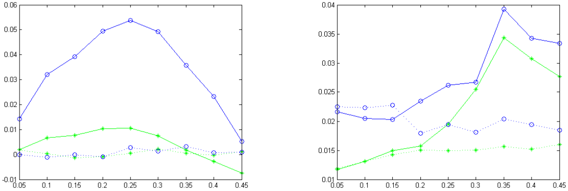
$$\operatorname{Var}\left(\frac{W^B_{\lambda,n}(u)}{\mathbb{E}(W^B_{\lambda,n}(u))}\right) \underset{n\to+\infty}{\sim} \frac{2v_{2,u}(H)}{C^2_{H,1}\, u^{4H}}\, n^{-1}.$$

Estimation on a fixed interval

$$\widehat{H}_n^F(u,v) = \frac{1}{2}\log\left(\frac{W^F_{\lambda,n}(u)}{W^F_{\lambda,n}(v)}\right) / \log\left(\frac{u}{v}\right) \text{ for } u \neq v$$

$$\widehat{H}^F_{n^\gamma}(u,v) \underset{n\to+\infty}{\longrightarrow} H \text{ a.s. if } \gamma > (1-2H)^{-1}$$

Gaussian case [Istas, Lang, 1997] for all $H \in (0,1)$, $\widehat{H}_n^B(u,v) \underset{n\to+\infty}{\longrightarrow} H$ a.s., with asymptotic normality if $H \in (0,3/4)$.



Bias $H - \widehat{H}_n(u,v)$ standard deviation

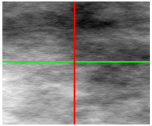
Figure: fPp (–) and fBm ($\cdot\cdot$) with $n = 2^{11}$, $\lambda = 1$, $(u,v) = (1,2)$ (o), $(u,v) = (1,4)$ (*) on 100 realizations.

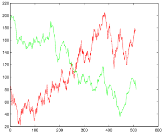
Application : fractal analysis in medical imaging

Goal : use fractal analysis to characterized self-similarity with a fractal index $H \in (0,1)$ and extract some helpfull informations for diagnosis

Numerous methods and studies ! [Lopes and Betrouni, 2009]

Quadratic variations method : image $(I(k_1, k_2))_{0 \le k_1,\, k_2 \le n-1}$

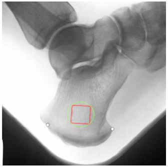




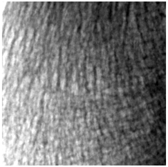
- **Extract** a line from the image $(L_\theta(k))_{0 \le k \le n_\theta - 1}$ for θ a direction.
- **Compute** $v_\theta(u) = \dfrac{1}{n_\theta - u} \displaystyle\sum_{k=0}^{n_\theta - 1 - u} (L_\theta(k+u) - L_\theta(k))^2.$
- **Average** along several lines of the same direction $\overline{v_\theta}(u)$ and compute $\widehat{H}_\theta(u,v) = \frac{1}{2} \log\left(\frac{\overline{v_\theta}(u)}{\overline{v_\theta}(v)}\right) / \log\left(\frac{u}{v}\right).$

Example : Bone Trabecular Micro-architecture

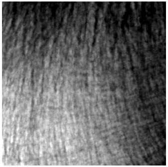
Data set : 211 numeric radiographs high-resolution of calcaneum (bone heel) standardized acquisition ROI 400 $\times$ 400 [Lespessailles et al., 2007] :



ROI



control case



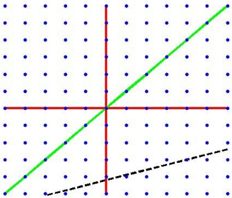
osteoporotic case

- Validation of self-similarity using power spectrum and variograms methods for calcaneous bone [Benhamou et al, 94], and cancellous bone [Caldwell et al, 94]
- Discrimination of osteoporotic cases [Benhamou et al, 2001]

$H_{mean} = 0.679 \pm 0.053$ (osteoporotic) $H_{mean} = 0.696 \pm 0.030$ (control)

Example : Bone Trabecular Micro-architecture

Implementation issues

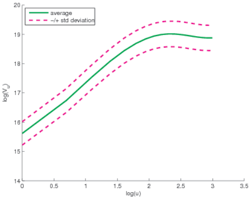


Black = out of lattice.
Precision of
red = 1, green = $\sqrt{2}$

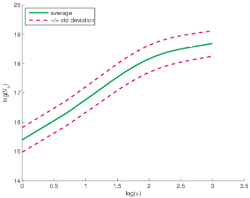
- Estimation on oriented lines **without interpolation**.
- Precision is not the same in all directions.
- Accuracy of orientation analysis $\leftrightarrow$ Precision of the image.

Example : Bone Trabecular Micro-architecture

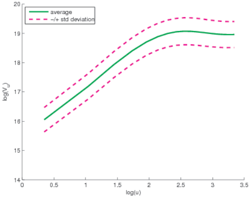
Bone radiographs (211 cases) : log-log plot of mean quadratic variations



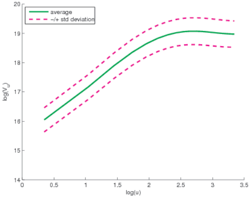
$\theta_1 = (1,0),\ H_{\theta_1} = 0.51 \pm 0.08$



$\theta_2 = (0,1),\ H_{\theta_2} = 0.56 \pm 0.06$



$\theta_3 = (1,1)/\sqrt{2},\ H_{\theta_3} = 0.51 \pm 0.08$

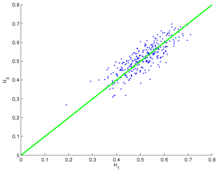


$\theta_4 = (-1,1)/\sqrt{2},\ H_{\theta_4} = 0.51 \pm 0.09$

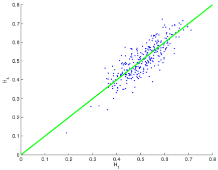
[Benhamou, HB, Richard, 2009]

Example : Bone Trabecular Micro-architecture

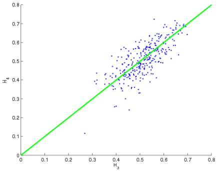
Comparison of the index in different directions



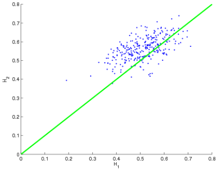
H_{θ_3} vs H_{θ_1}



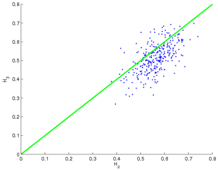
H_{θ_4} vs H_{θ_1}



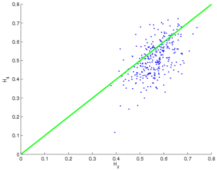
H_{θ_4} vs H_{θ_3}



H_{θ_2} vs H_{θ_1}



H_{θ_3} vs H_{θ_2}

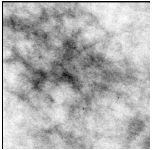


H_{θ_4} vs H_{θ_2}

$1 : \theta_1 = (1, 0)$ (horizontal), $2 : \theta_2 = (0, 1)$ (vertical),
$3 : \theta_3 = (1, 1)/\sqrt{2}$ (diagonal), $4 : \theta_4 = (-1, 1)/\sqrt{2}$ (diagonal).

Example : Mammograms

dense breast tissue



fatty breast tissue

- Validation of self-similarity using a power spectrum method [Heine et al, 2002]

$$H \in [0.33, 0.42].$$

- [HB, Richard, 2010] using variogram method on 58 cases with 2 mammograms ROI 512×512

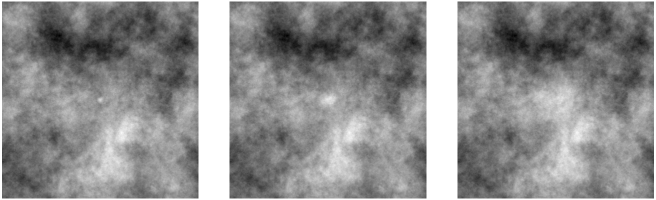
$$H = 0.31 \pm 0.05$$

- Discrimination of dense and fatty breast tissue using a wavelet method (WTMM) [Kestener et al, 2001]

$H \in [0.55, 0.75]$ (dense tissues) $H \in [0.2, 0.35]$ (fatty tissues)

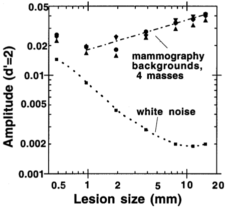
Spot detection on mammograms

Simulated spot with identical contrast on a mammogram [Grosjean, Moisan, 2009]

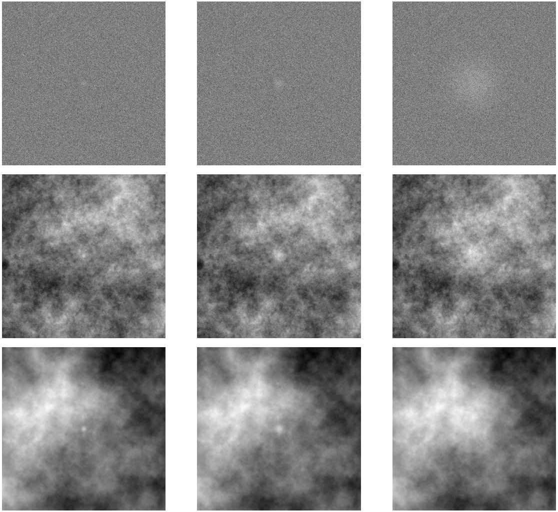


Link between size and contrast for spot detection

Burgess' law [Burgess et al, 2001]



Simulated spot with identical contrast on simulated fields 512 $\times$ 512



White noise

$H = 0.3$

$H = 0.7$

radius 5 | radius 10 | radius 50

References

- C.L. Benhamou, H. Biermé and F. Richard (2009) : Analysis of texture anisotropy based on some Gaussian fields with spectral density. *Proc. MICCAI, 12th Int. Conference, London.*
- H. Biermé, Y. Demichel and A. Estrade (2013) : Fractional Poisson field and Fractional Brownian field : why are they resembling but different ? *ECP,* **18***(11),1-13.*
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