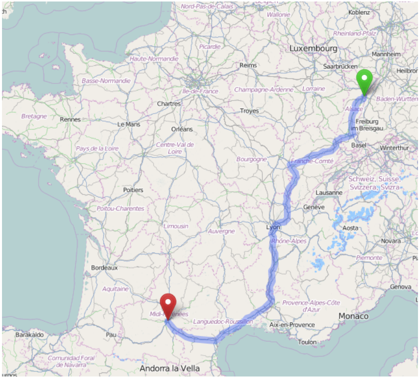
Improper Poisson line process as a SIRSN

Jonas KAHN

Nantes
April 6th, 2016

Motivation



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Andorra la Vella

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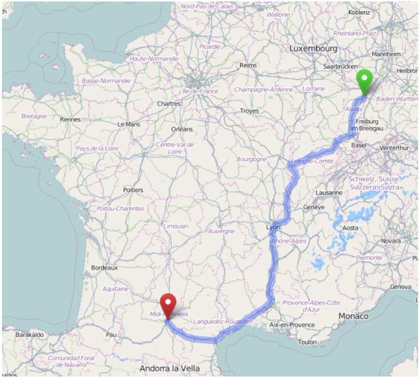

- Invariant by translation, rotation

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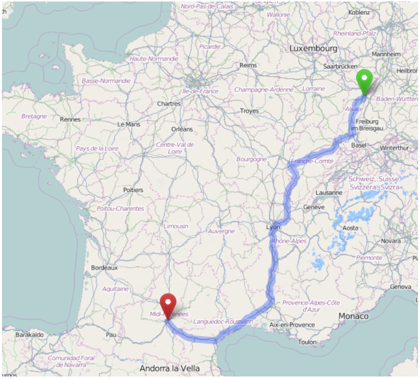

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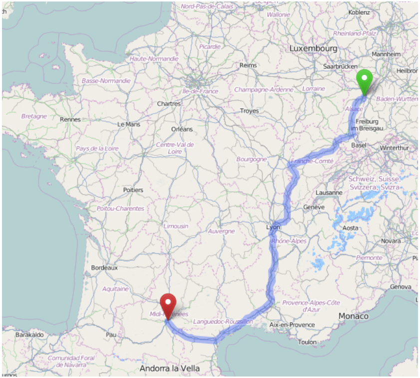

- Invariant by translation, rotation, and scaling
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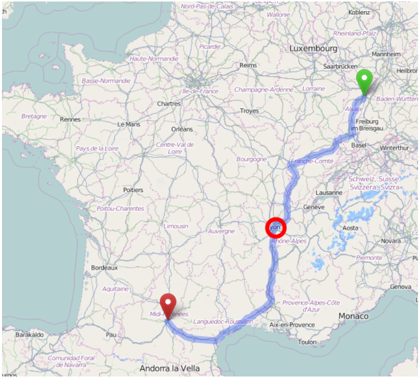


- Invariant by translation, rotation, and scaling
- Routes are foremost, compatible

SIRSN : Scale-invariant random spatial network

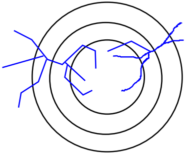
A SIRSN is a set of finite routes (paths) in $\mathbb{R}^d$, such that :

1. $\forall x_1, x_2 \in \mathbb{R}^d$, there is a.s. a unique route $\mathcal{R}(x_1, x_2)$.
2. If $x_1, \ldots, x_k \in \mathbb{R}^d$ the network $\mathcal{N}(x_1, \ldots, x_k)$ of routes between each pair of x_i is invariant by all similarities $\mathcal{S}$: the networks $\mathcal{N}(\mathcal{S}x_1, \ldots, \mathcal{S}x_k)$ and $\mathcal{S}\mathcal{N}(x_1, \ldots, x_k)$ have the same distributions.
3. The length D_1 of the route between 0 and 1 has finite expectation : $\mathbb{E}\left[D_1\right] < \infty$.
4. Let $\Xi = \bigcup_{i \in \mathbb{N}} \Xi_i$ where the Ξ_i are independent Poisson processes with intensity 1. The following long-distance network has finite intensity $p(1)$:

$$\bigcup_{x_1, x_2 \in \Xi} \left(\mathcal{R}(x_1, x_2) \setminus \left(B(x_1, 1) \cup B(x_2, 1)\right)\right).$$

Quelques propriétés (Aldous)

There are motorways :



There are singly-infinite “geodesics”', but no doubly-infinite “geodesic”.

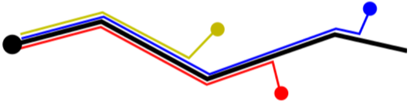


Figure : Singly-infinite

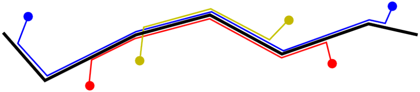


Figure : Doubly-infinite

Heuristics for switching from tables to Dijkstra after $p^{\frac{2}{3}}(1)M^{\frac{1}{3}}$ nodes.

Does it exist ?

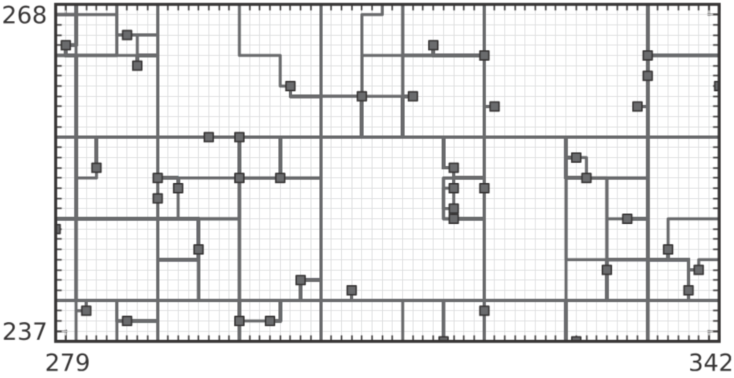


Fig. 4. The spanning subnetwork (within a rectangular window) on sampled points (■) in a discrete approximation to model 1.

Figure from Aldous and Ganesan (PNAS 2013)

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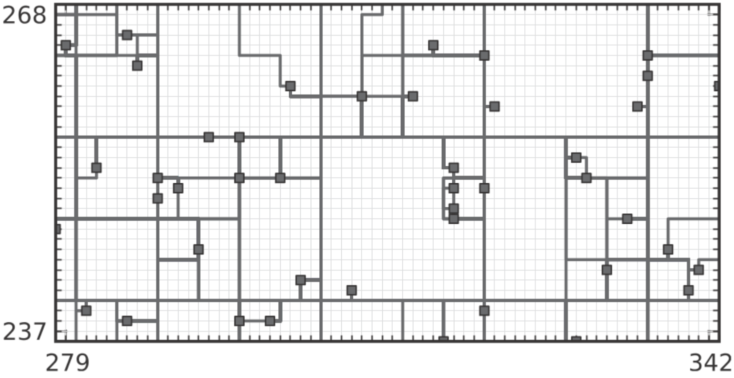

Fig. 4. The spanning subnetwork (within a rectangular window) on sampled points (■) in a discrete approximation to model 1.

Figure from Aldous and Ganesan (PNAS 2013)

Not very satisfying : Similarity invariance has to be added *a posteriori*.

Poisson line process I

Poisson point process

Let Λ a measure on $\mathbb{X}$. A Poisson point process is a random set of points in $\mathbb{X}$ such that, denoting $N(B)$ the number of points in B :

- For any disjoint measurable B_i, the random variables $N(B_i)$ are disjoint.
- $N(B)$ is a Poisson random variable with parameter $\Lambda(B)$.

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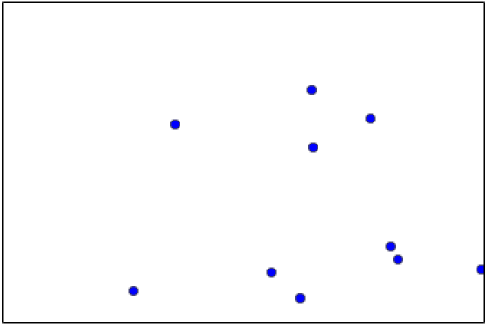

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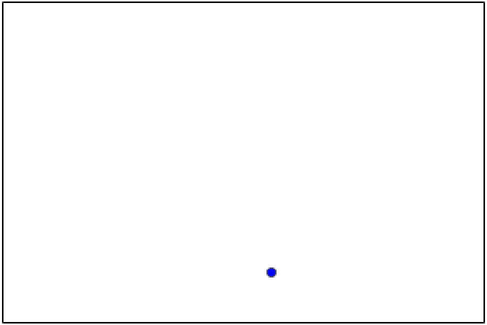

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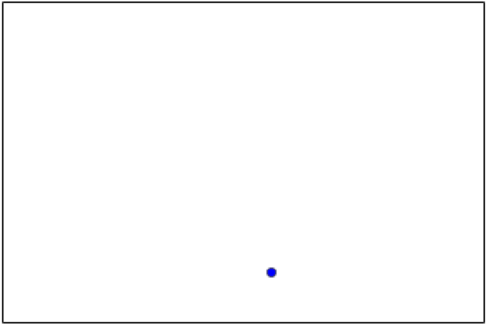

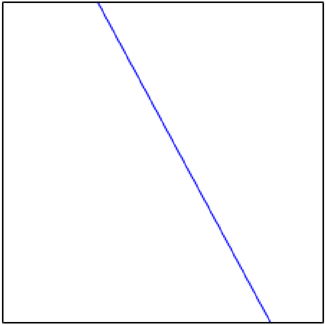

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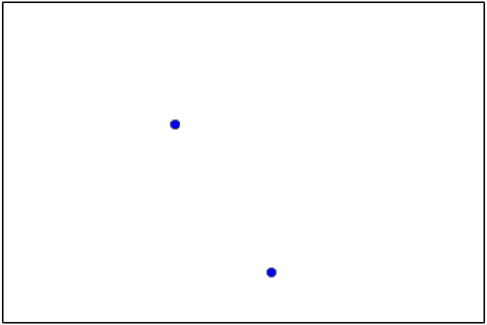

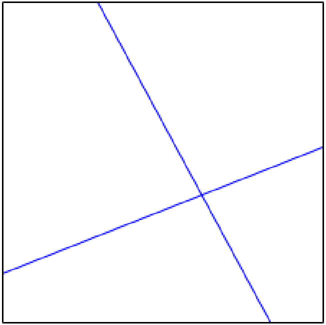

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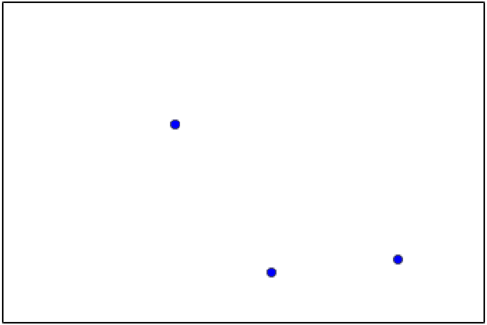

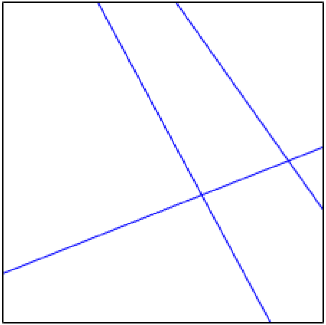

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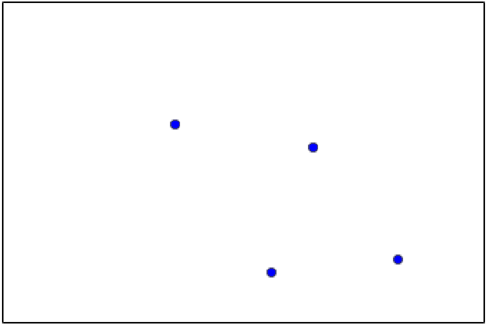

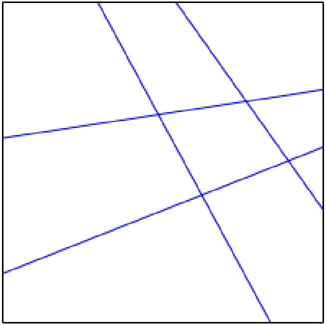

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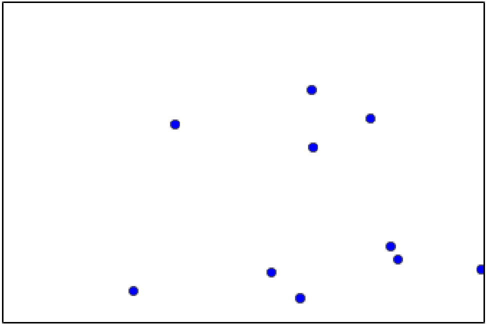

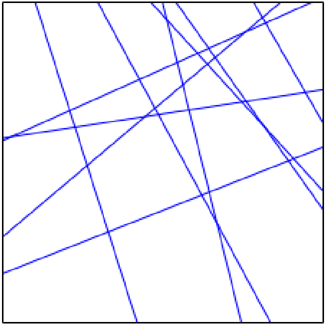

Poisson line process II

Parametrisation of lines

- Direction. In 2d, angle with axis of abscissas.
- Intersection with normal hyperplane. In 2d, algebraic distance to the origin.

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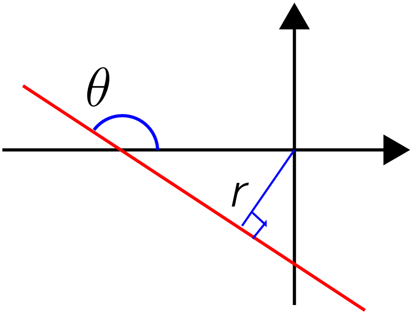

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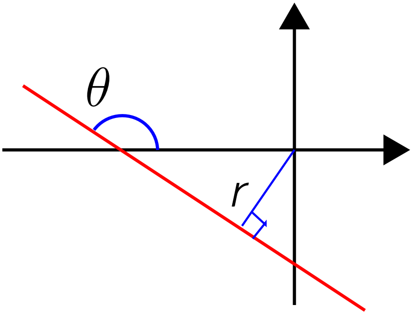

$\Longrightarrow$ Image of a homogeneous PPP is translation and rotation invariant

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Under this distribution, the number of lines hitting a convex K is a Poisson variable with parameter the perimeter of K (hyperarea in dimension > 2) :

$$\mu_d([K]) = \text{hyperarea of the boundary of } K$$

Poisson line process III

Let's add a dimension to the PPP : a speed limit on each line.
Measure of PPP :

$$(\gamma - 1)v^{-\gamma}\mathrm{d}v\mu_d(\mathrm{d}l)$$

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The number of lines faster than v_0 hitting a convex K is a Poisson variable with parameter :

$$\lambda = \pi_d([K] \cap \{v \geq v_0\}) = {v_0}^{-(\gamma-1)} \cdot \text{hypersurface de } K$$

Poisson line process IV

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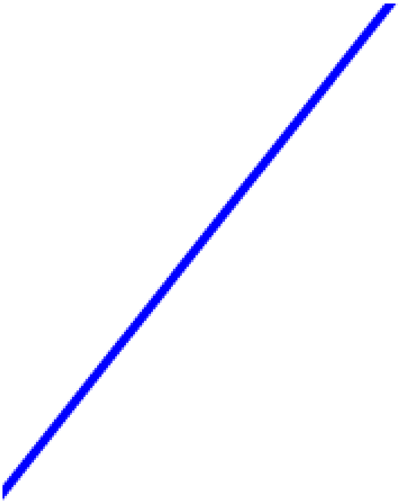

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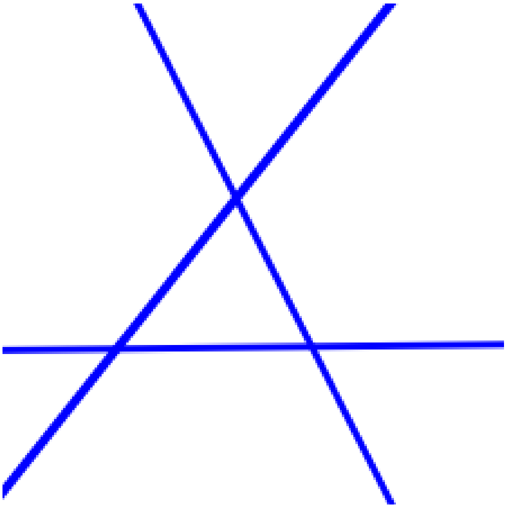

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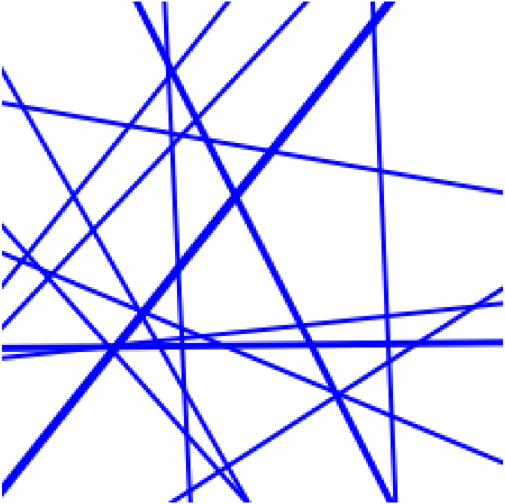

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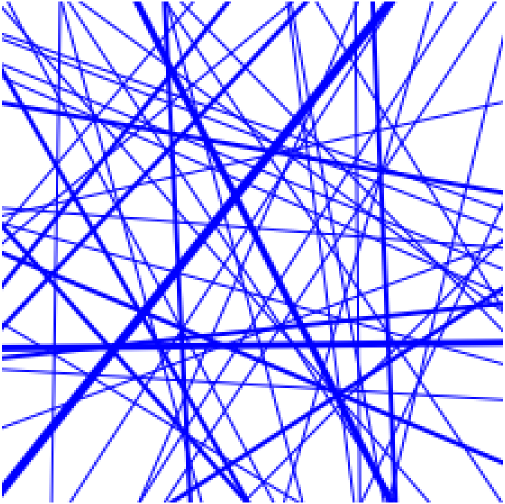

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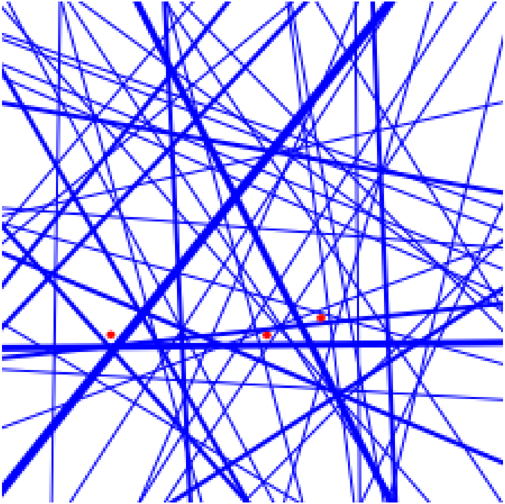

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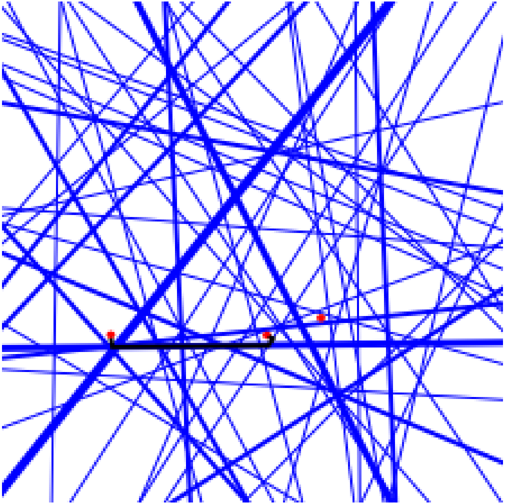

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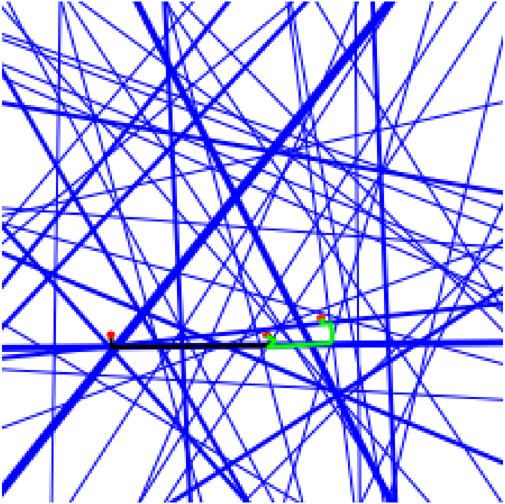

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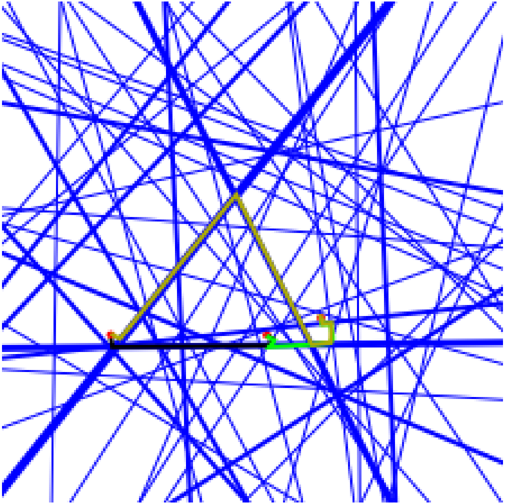

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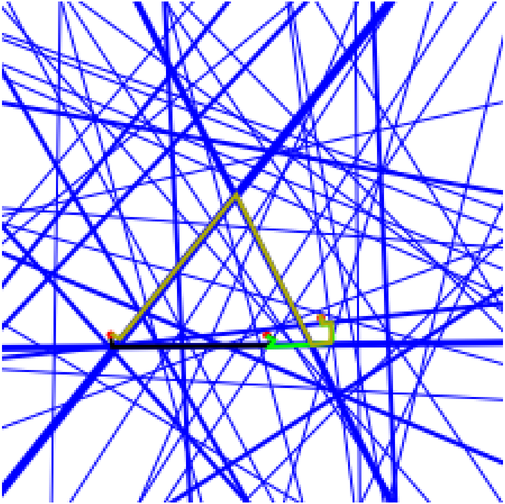

SIRSN ?

SIRSN : Scale-invariant random spatial network

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Time diameter of balls

Let T_{xy} be the minimum time to go from x to y while respecting speed limits.

Theorem (idea from Kendall)

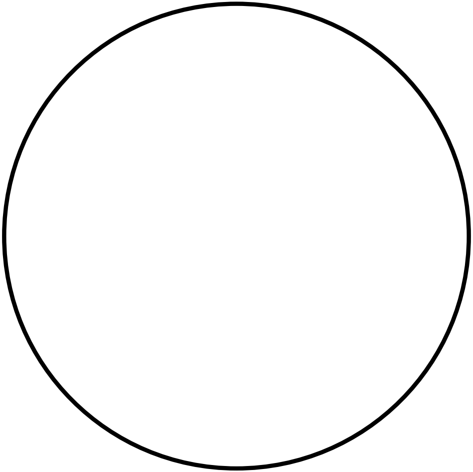
Let $\gamma > d \geq 2$.
Let B a ball with radius r. There is T_1 such that for all $\frac{1}{2} > \varepsilon > 0$, with probability at least $1 - \varepsilon$:

$$\begin{aligned} T_B \hat{=} \sup_{x,y \in B} T_{xy} \\ \leq T_1 r^{\frac{\gamma - d}{\gamma - 1}} \left(\ln \frac{1}{\varepsilon} \right)^{1/(\gamma - 1)} \end{aligned}$$

Notably, this maximum time has the following moment, for all $\delta < T_1^{-\frac{1}{\gamma-1}} r^{-\frac{d-1}{\gamma-1}}$:

$$\mathbb{E}\left[\exp\left(\delta T_B^{\gamma-1}\right)\right] < \infty.$$

Construction



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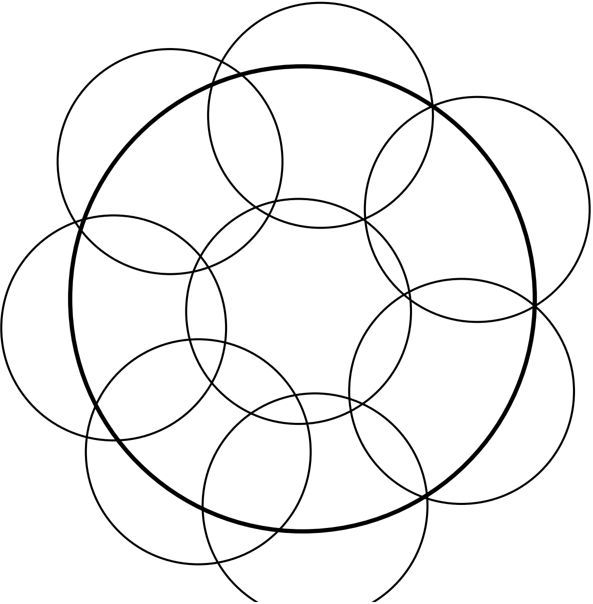
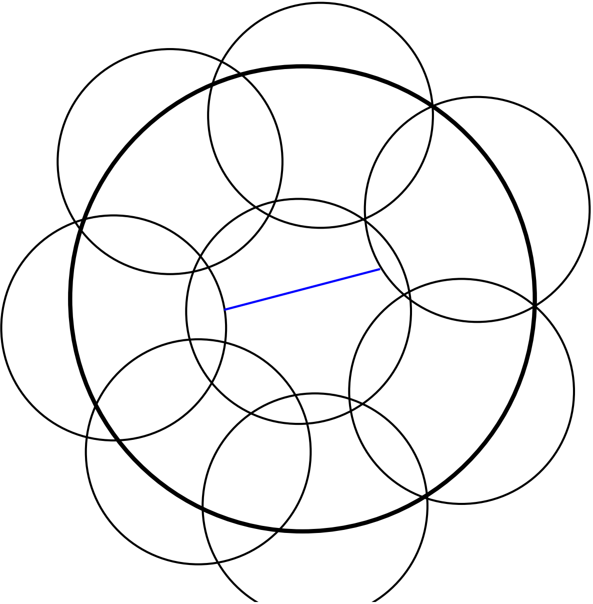


α^{-1}

Construction



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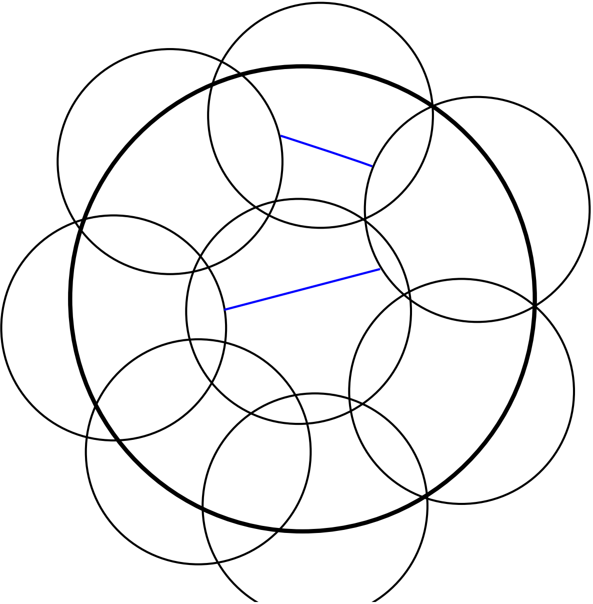

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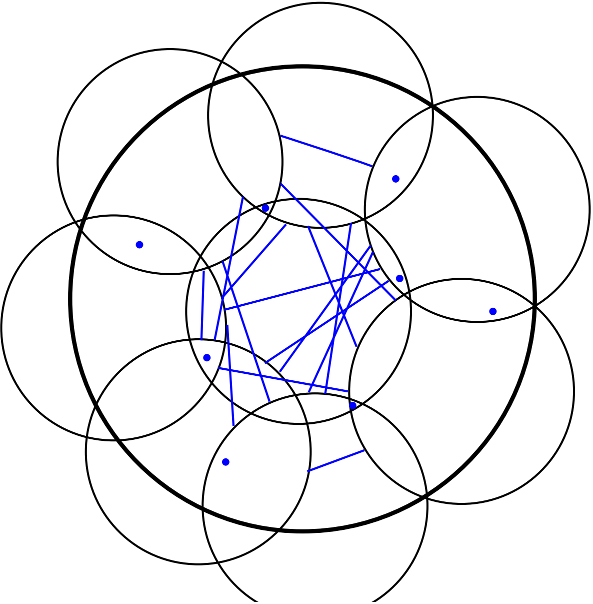

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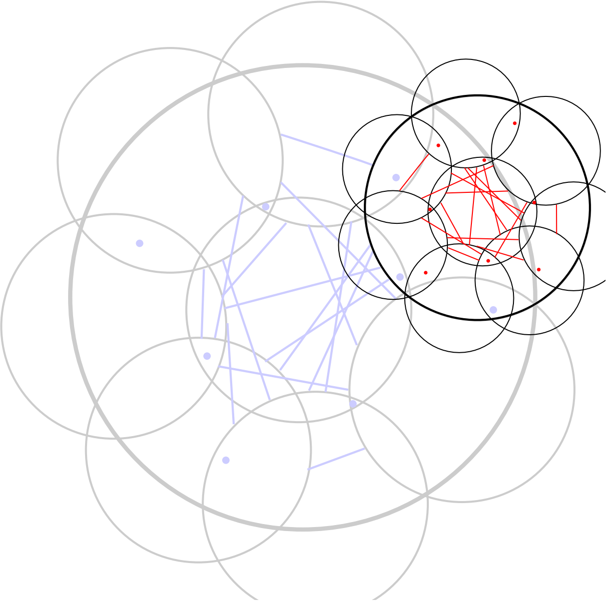

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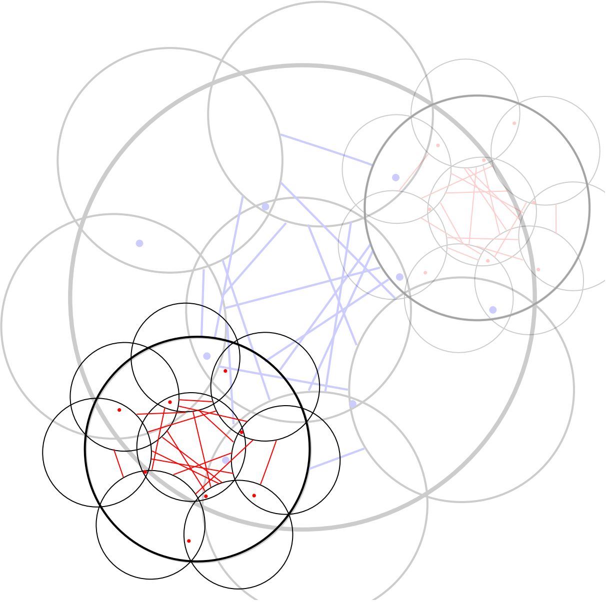

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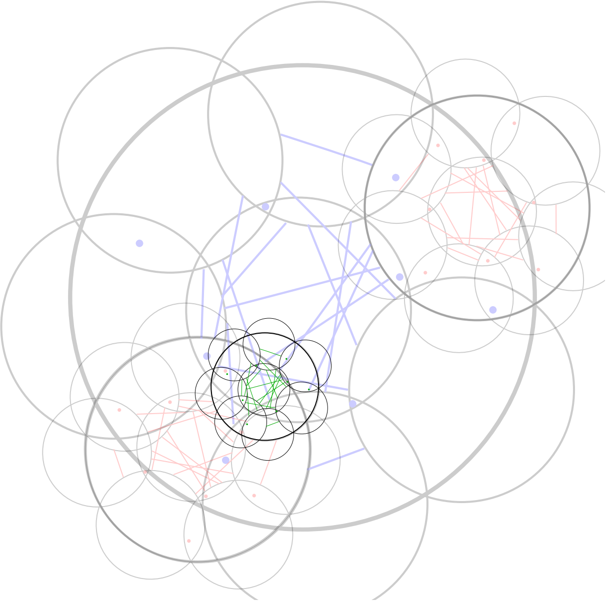

α^{-n}

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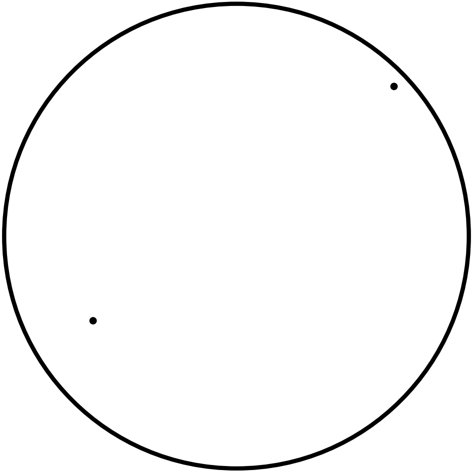

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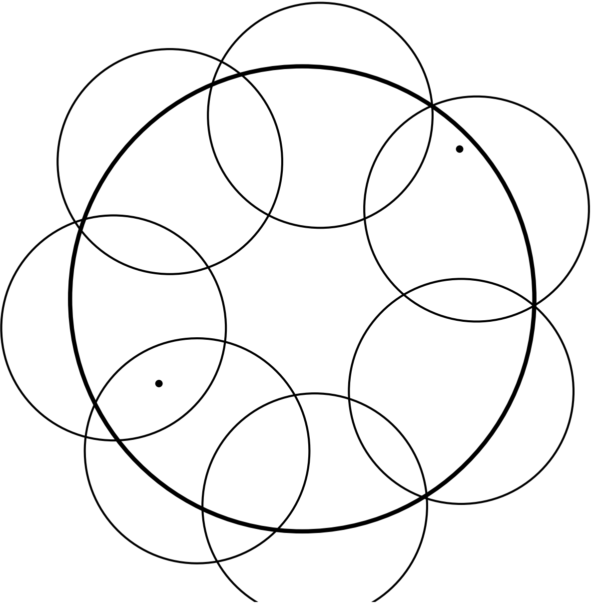

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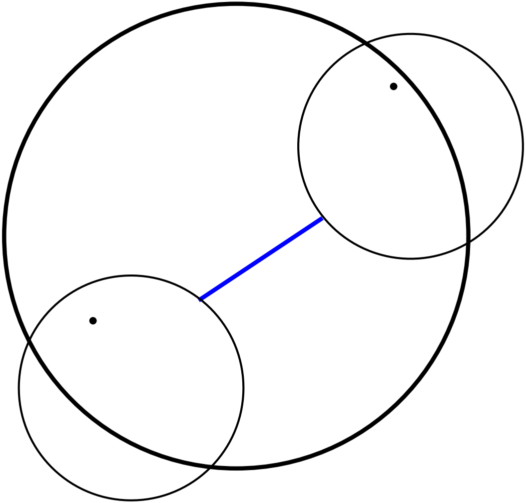

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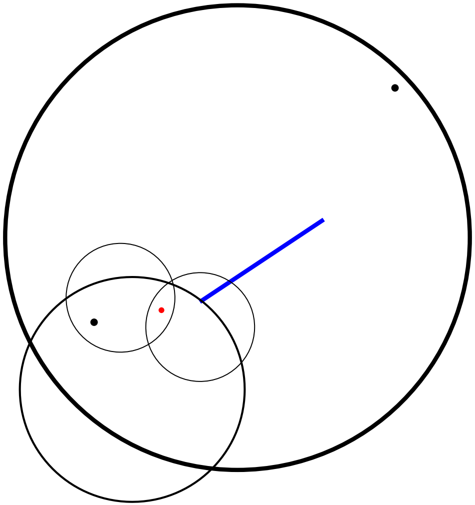

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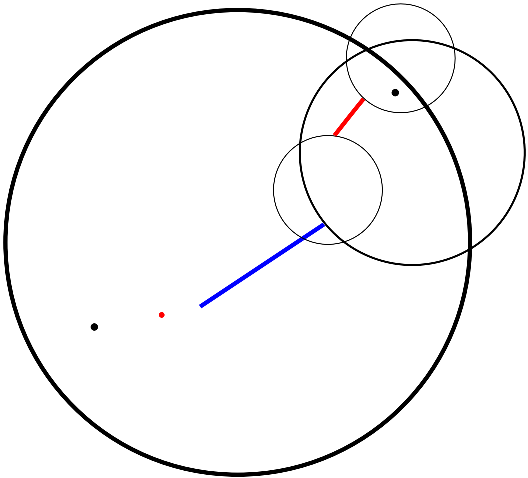

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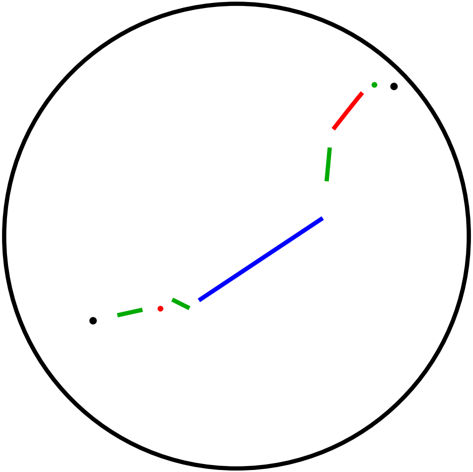

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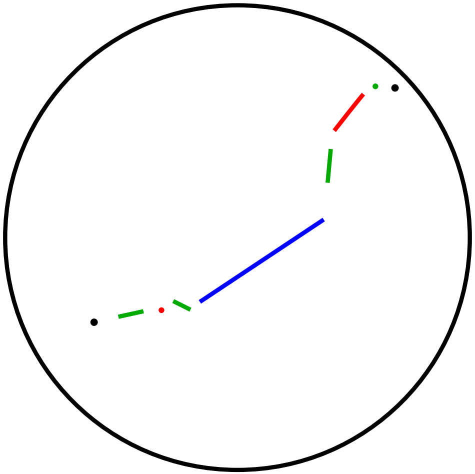
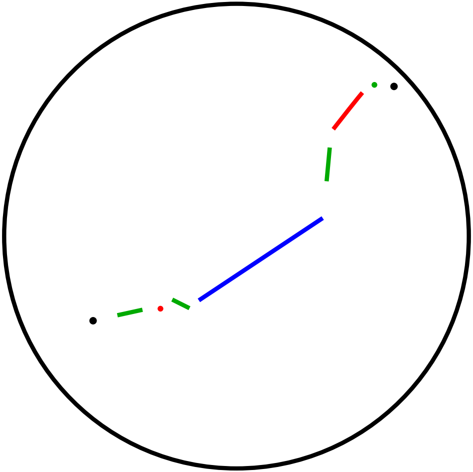


α^{-n}

$$\lambda \equiv \frac{r^{d-1}}{v^{\gamma-1}}$$

Construction



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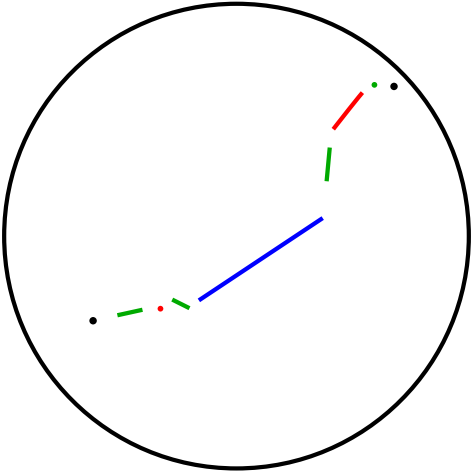

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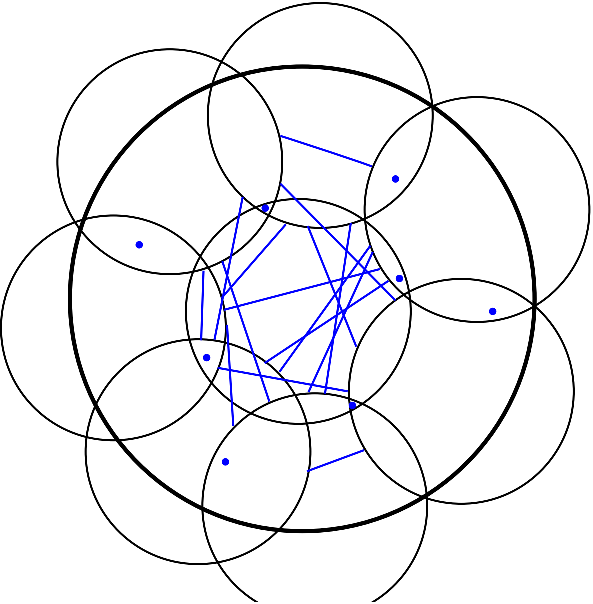

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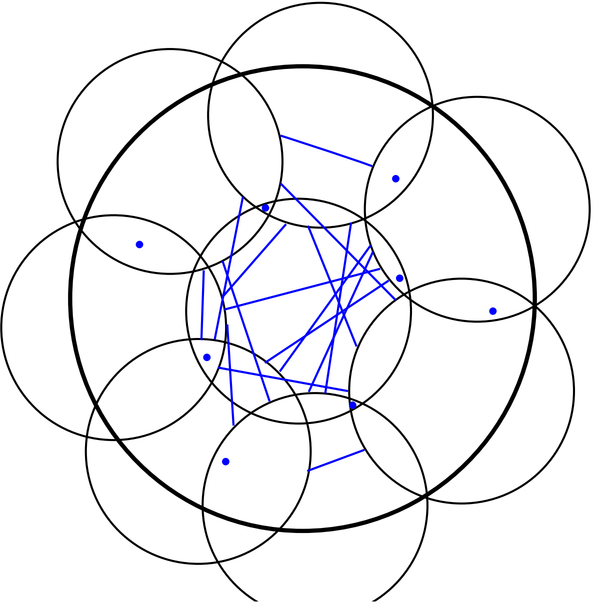

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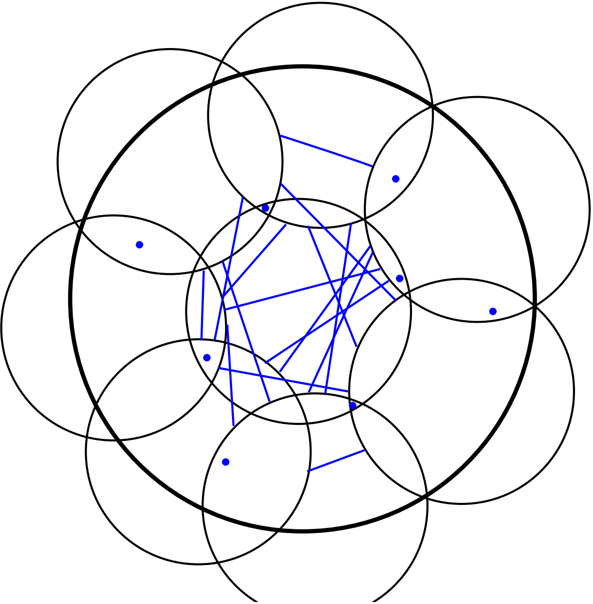

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$$\sum (2\alpha)^{d(n+1)} e^{-\lambda_n} \leq \varepsilon$$

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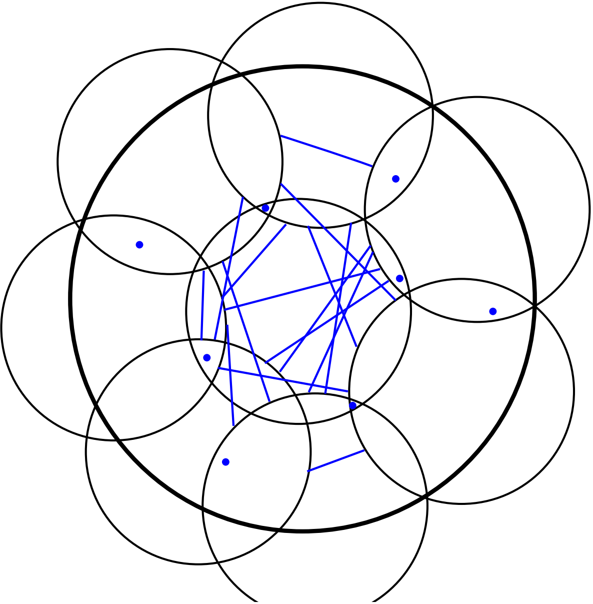

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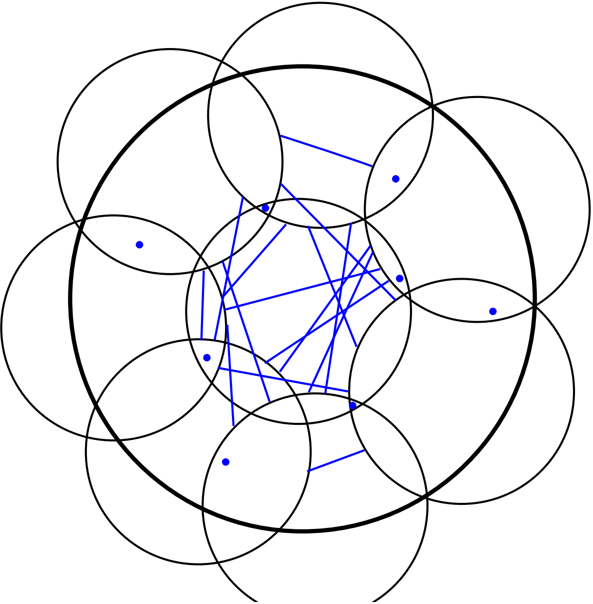

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Consequence

Our Poisson line process generates a random metric space.

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Theorem (Kendall)

The minimum time to connect each pair of points is attained. There is at least one geodesic between each pair of points.

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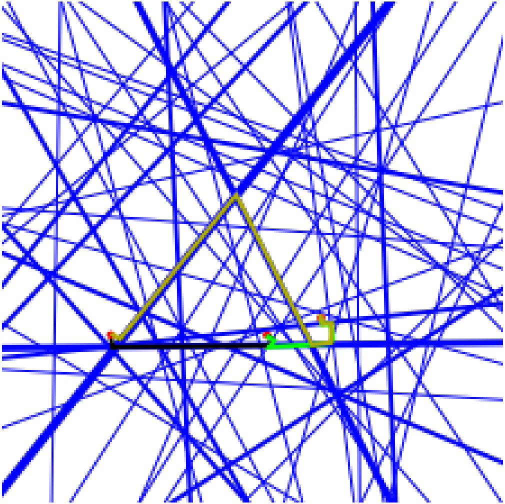
The metric is given by the time needed to travel between two points.

Poisson line process IV



SIRSN : Scale-invariant random spatial network

A SIRSN is a set of finite routes (paths) in $\mathbb{R}^d$, such that :

1. $\forall x_1, x_2 \in \mathbb{R}^d$, there is a.s. a unique route $\mathcal{R}(x_1, x_2)$.
2. If $x_1, \ldots, x_k \in \mathbb{R}^d$ the network $\mathcal{N}(x_1, \ldots, x_k)$ of routes between each pair of x_i is invariant by all similarities $\mathcal{S}$: the networks $\mathcal{N}(\mathcal{S}x_1, \ldots, \mathcal{S}x_k)$ and $\mathcal{S}\mathcal{N}(x_1, \ldots, x_k)$ have the same distributions.
3. The length D_1 of the route between 0 and 1 has finite expectation : $\mathbb{E}[D_1] < \infty$.
4. Let $\Xi = \bigcup_{i \in \mathbb{N}} \Xi_i$ where the Ξ_i are independent Poisson processes with intensity 1. The following long-distance network has finite intensity $p(1)$:

$$\bigcup_{x_1, x_2 \in \Xi} \left(\mathcal{R}(x_1, x_2) \setminus (B(x_1, 1) \cup B(x_2, 1))\right).$$

Mean length of a geodesic

Theorem

The length D_1 between 0 and 1 has a finite mean :

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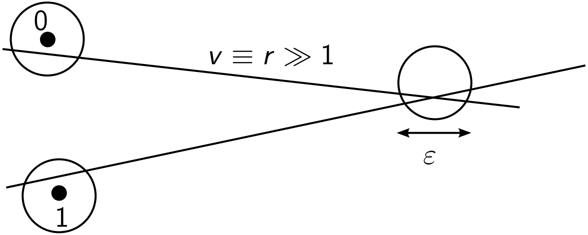
Conjecture

D_1 has a δ-moment if and only if $\delta < 2\gamma + d - 3$.



Geodesics are unique

Theorem (Kendall in dimension 2)

For all $d \geq 2$, for any pair of points x and y in $\mathbb{R}^d$, the geodesic g_{xy} is almost surely unique.

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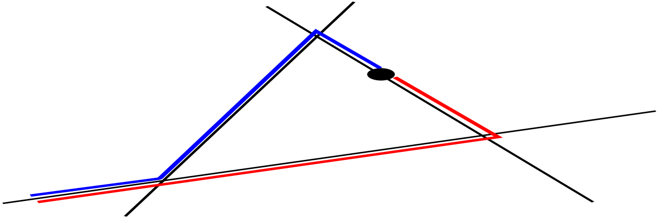
Remark

Almost surely, there are pairs of points x and y in $\mathbb{R}^d$ with several geodesics.



Many directions

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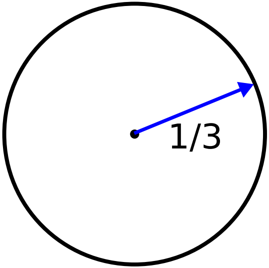
Intensity



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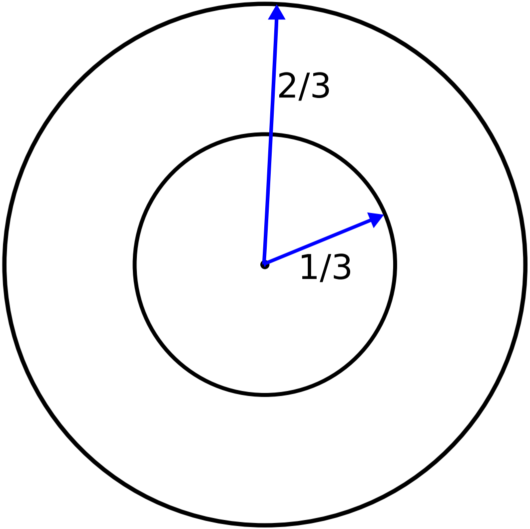

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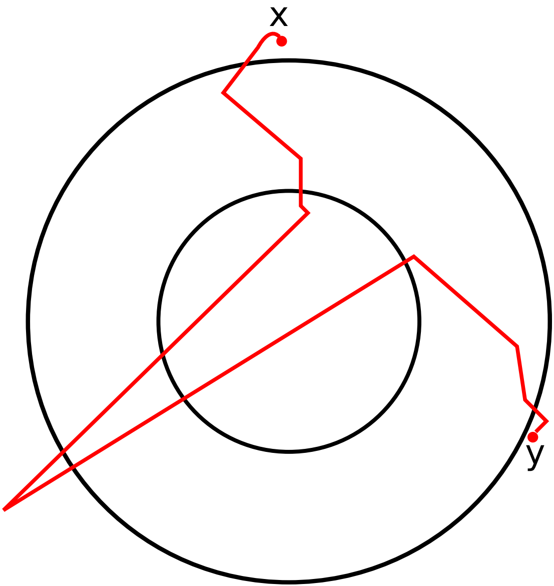

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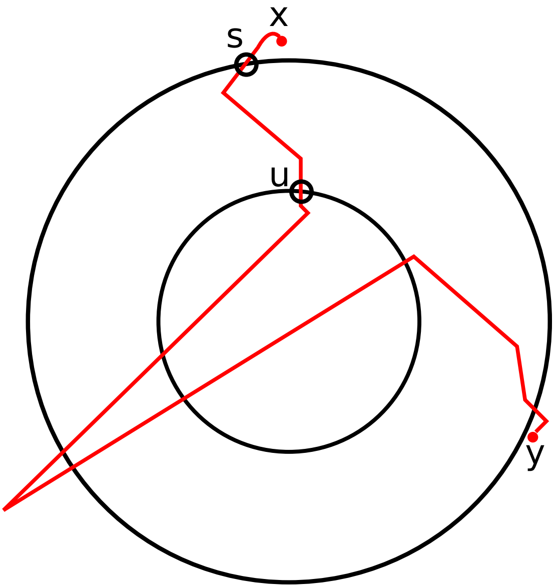

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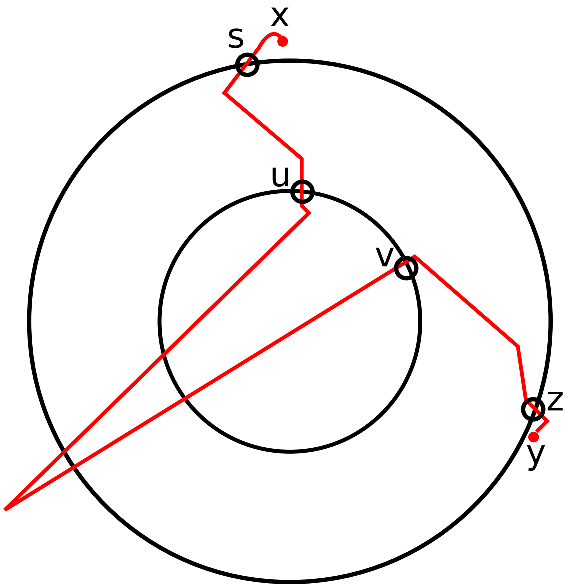

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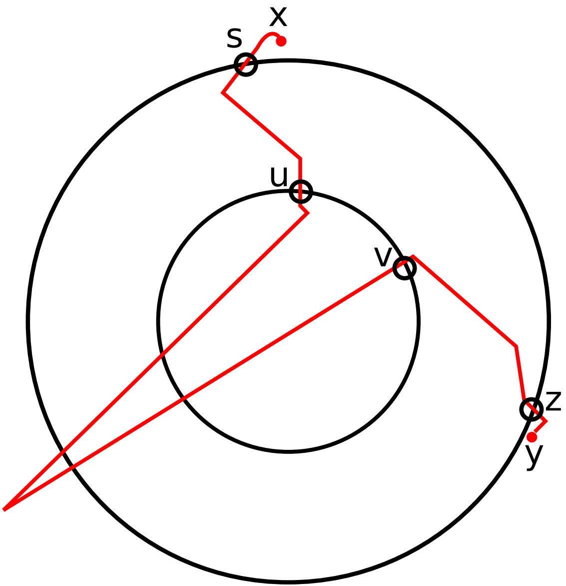

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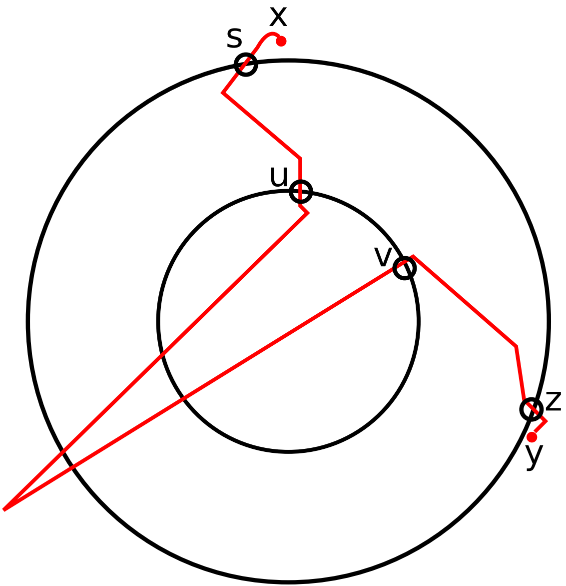

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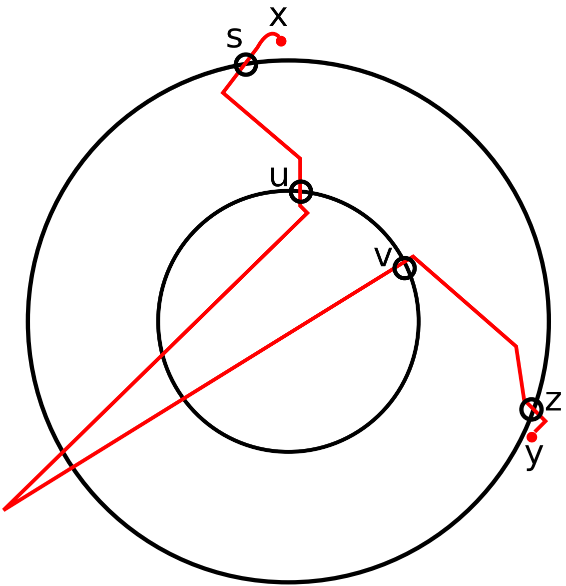

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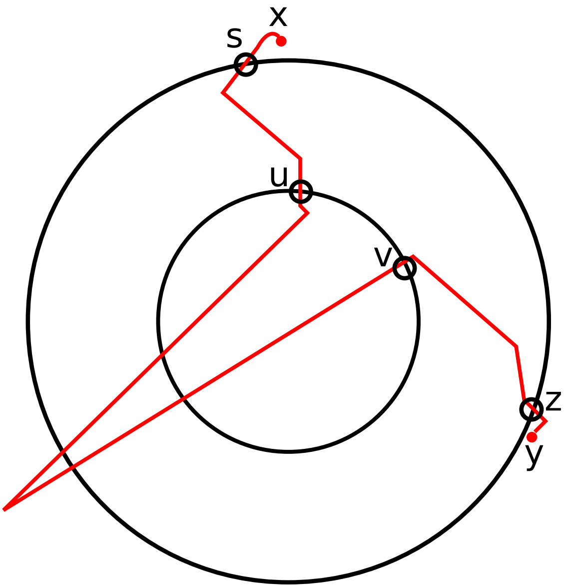

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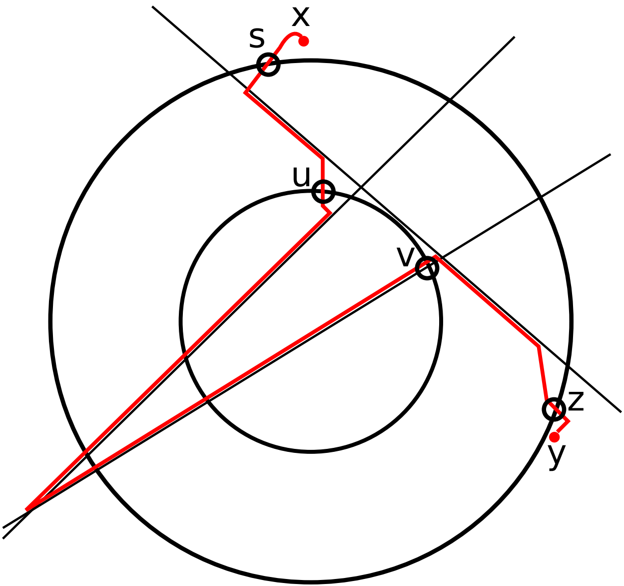

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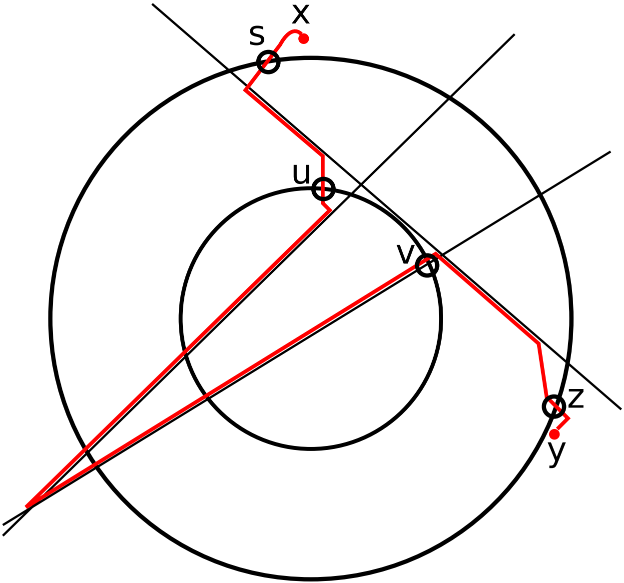

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$$\mathcal{V}_\varepsilon$$

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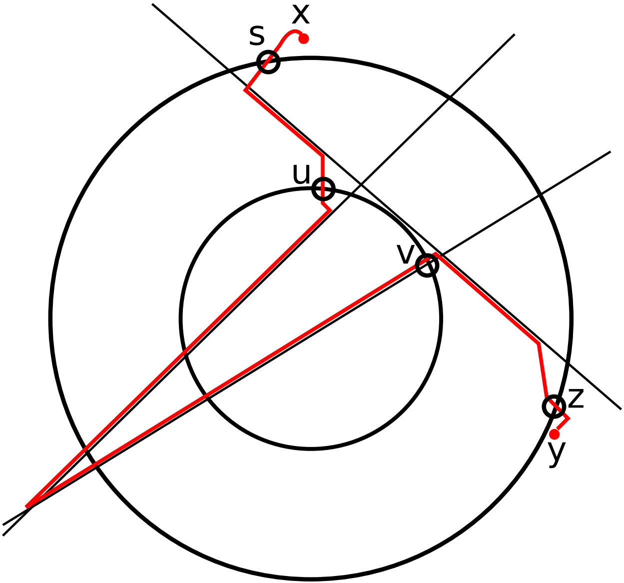

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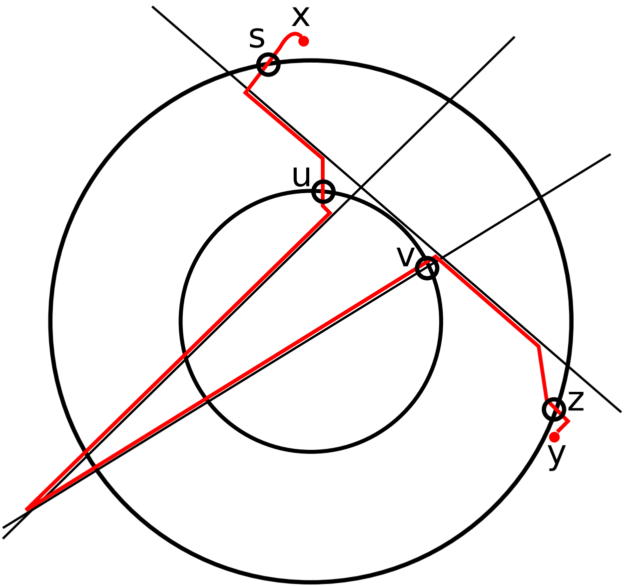

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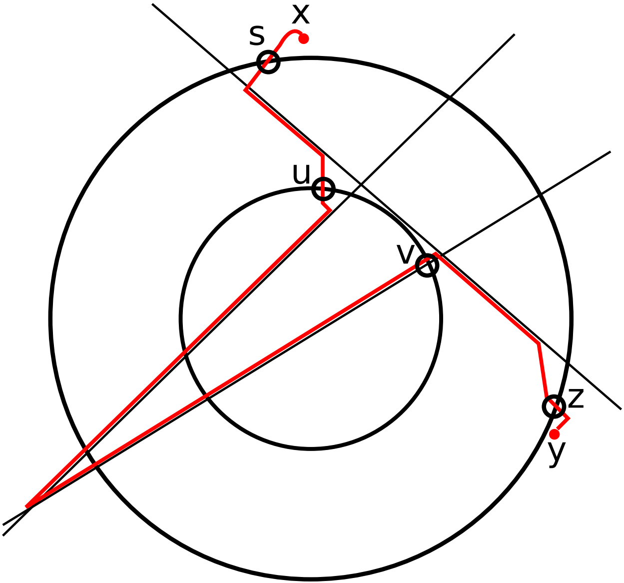

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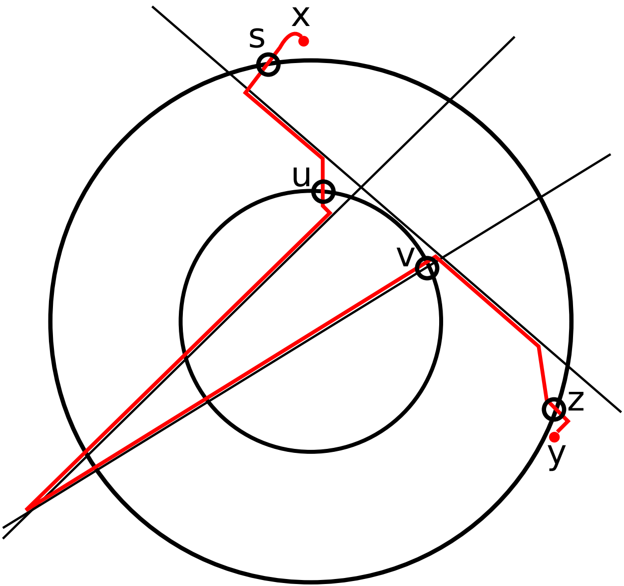

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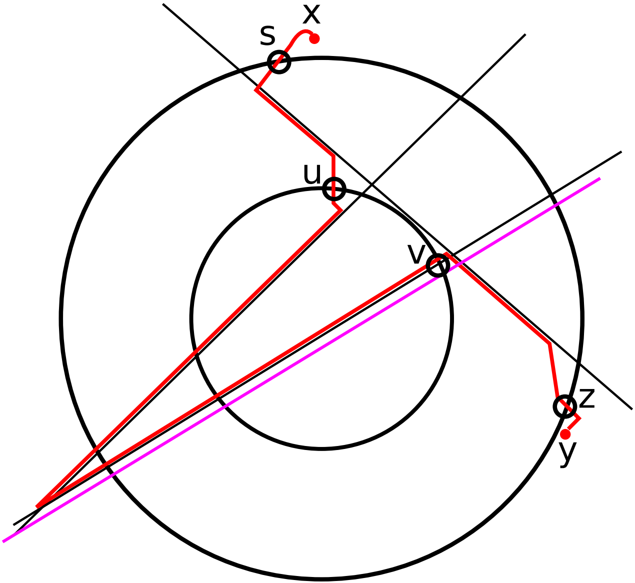

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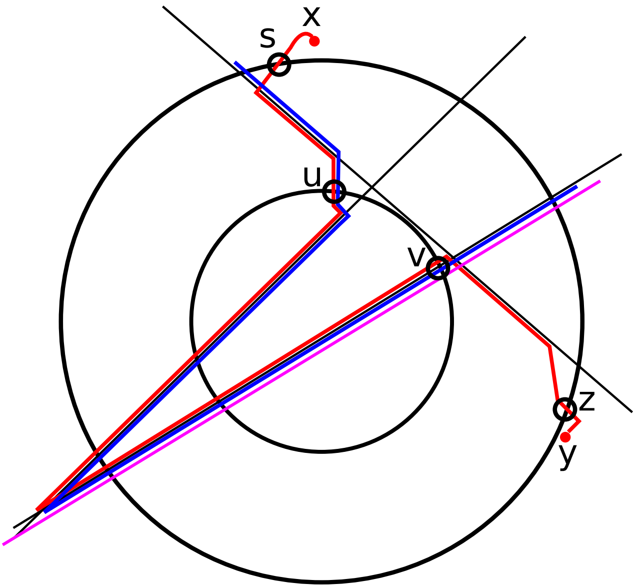

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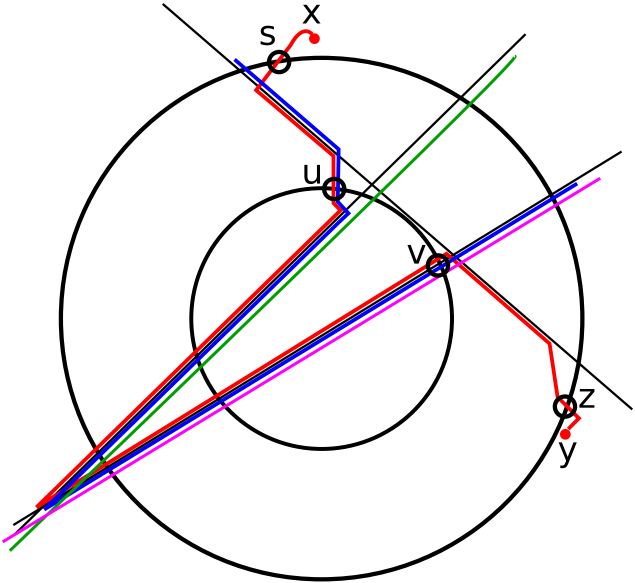

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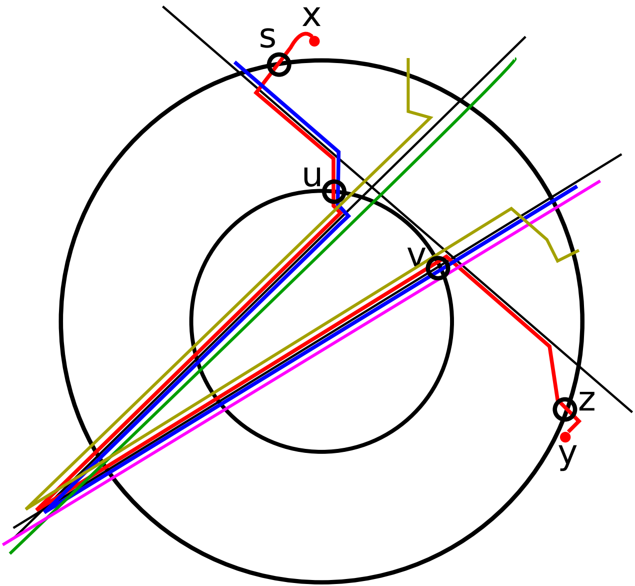

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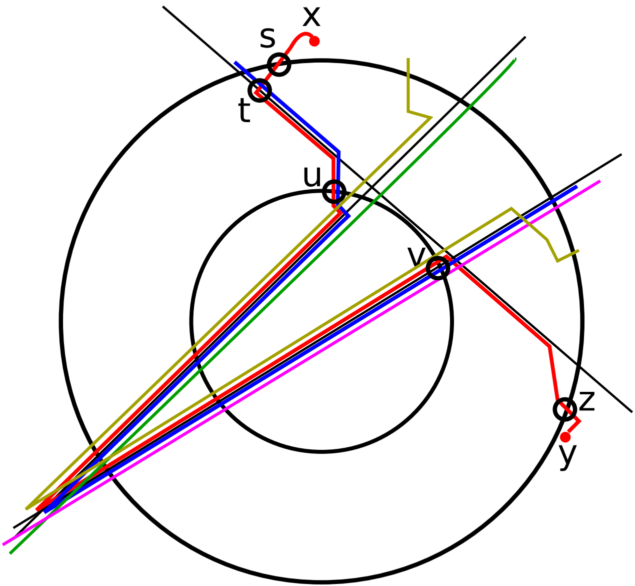

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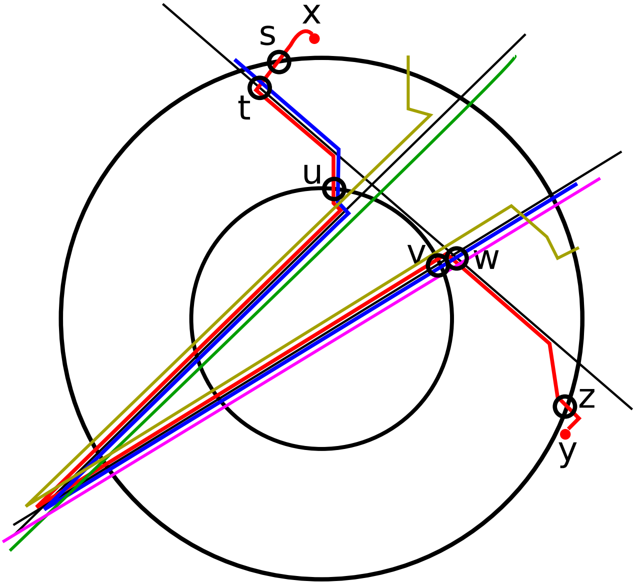

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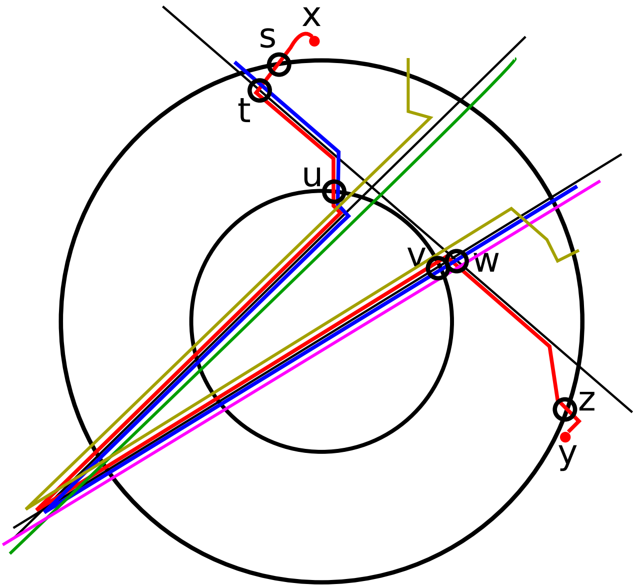

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$$\exp\left(\delta\sqrt{\mathcal{L}}\right) < \infty$$

Bonus : compare with Brownian map

The Brownian map is a random metric space :

- homeomorphic to the dimension 2 sphere.
- with Hausdorff dimension 4.
- whose geodesics without their extremal points span a set of Hausdorff dimension 1.
- whose cut-locus from a point has Hausdorff dimension 2 and is a tree.

Our random metric space :

- is homeomorphic to $\mathbb{R}^d$, with dimension d.
- has Hausdorff dimension $\frac{d\gamma - d}{\gamma - d} > d$. For $d = 2$ and $\gamma = 3$, we get 4.
- (under the hypothesis that any geodesic is a limit of geodesics between points in a dense set) whose geodesics without their extremal points span a set of Hausdorff dimension 1.
- cut-locus ?