Multi-dimensional backward stochastic differential equations of diagonally quadratic generators: the general result

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Outline

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Fix a terminal time $T \in (0,+\infty)$ and two positive integers n and d. Let $(\Omega, \mathcal{F}, (\mathcal{F}_t)_{t\in[0,T]}, \mathbb{P})$ be a complete filtrated probability space, and $(B_t)_{t\in[0,T]}$ be a d-dimensional B.M. generating $(\mathcal{F}_t)_{t\geq 0}$.

We consider the following multi-dimensional backward stochastic differential equation (BSDE for short):

$$Y_t = \xi + \int_t^T g(s, Y_s, Z_s)\mathrm{d}s - \int_t^T Z_s \mathrm{d}B_s, \quad t \in [0,T], \tag{1.1}$$

where

- the terminal value $\xi \in \mathcal{F}_T$ is an n-dimensional random vector,
- the generator of BSDE (1.1) is the random function
  $$g(\omega, t, y, z): \ \Omega \times [0,T] \times \mathbb{R}^n \times \mathbb{R}^{n\times d} \to \mathbb{R}^n,$$
  which is $(\mathcal{F}_t)$-progressively measurable for each $(y,z) \in \mathbb{R}^n \times \mathbb{R}^{n\times d}$,
- a solution of BSDE (1.1) is a pair of $(\mathcal{F}_t)$-P.M. processes $(Y_t, Z_t)_{t\in[0,T]}$ with values in $\mathbb{R}^n \times \mathbb{R}^{n\times d}$ which almost surely verifies BSDE (1.1).

The history of BSDEs (1.1) can be dated back to

- linear case: Bismut (1973,JMAA);
- matrix-valued nonlinear case, quadratic form: Bismut (1976,SICON);
- uniformly Lipschitz generator: Pardoux and Peng (1990,SCL).

There has been an increasing interest in BSDEs with applications in various fields such as

- stochastic control,
- mathematical finance,
- partial differential equations (PDEs).

The class of BSDEs, with generators having a quadratic growth in the state variable z, has attracted a lot of attention in recent years.

The existence and uniqueness theory is well developed in the scalar case.

- the first existence and uniqueness result for scalar-valued quadratic BSDEs with bounded terminal values: Kobylanski (2000,AP);
- some subsequent intensive efforts
  - ◇ for the bounded terminal value case:
    Tevzadze (2008,SPA), Briand and Elie (2013,SPA),
    Fan (2016,SPA), Luo and Fan (2018,SD), etc
  - ◇ for the unbounded terminal value case:
    Briand and Hu (2006,PTRF; 2008,PTRF),
    Delbaen, Hu and Richou (2011,AIHPPS; 2015,DCDS),
    Barrieu and El Karoui (2013, AP),
    Fan, Hu and Tang (2020, CRM), etc.

For multidimensional quadratic BSDE:

- a counterexample:

  Frei and Dos Reis (2011,MFE),
  a simple generator and a bounded terminal value,
  fail to have a global bounded solution on $[0, T]$;

- some tools used when $n = 1$ can no longer be applied when $n > 1$ in most cases, like

  - Girsanov's transform,
  - comparison theorem,
  - monotone convergence, etc.

Consequently, multidimensional quadratic BSDEs pose a great challenge. Solutions of multidimensional quadratic BSDEs with unbounded terminal values have been listed as an open problem in Peng (1999, page 270).

Nevertheless, motivated by their intrinsic mathematical interest and especially by diverse applications in various fields, such as

- nonzero-sum risk-sensitive stochastic differential games,
- financial price-impact models,
- financial market equilibrium problems for several interacting agents,
- stochastic equilibria problems in incomplete financial markets,

many scholars have studied systems of quadratic BSDEs in recent years.

First of all, by the theory of BMO martingales and using the contract mapping argument, Tevzadze (2008,SPA) proved the first general existence and uniqueness result for multi-dimensional quadratic BSDEs

when the terminal value is small enough in the supremum norm, which has inspired subsequent works under some different types of "smallness" assumptions on the terminal value and the generator, see for example

- Frei (2014,SPA); Kardaras (2015,ArXiv);
- Jamneshan, Kupper and Luo (2017,ECP);
- Kramkov and Pulido (2016,AAP; 2016,SIAM);
- Harter and Richou (2019,EJP).

We also note that some different ideas and methods have been applied in these works mentioned above. Secondly, in the Markovian setting,

- Cheridito (2015,Stochastics) proved the solvability for a special system of quadratic BSDEs,
- Xing and Zitkovic (2018,AP) obtained, by virtue of analytic PDE methods, the global solvability for a large class of multidimensional quadratic BSDEs with weak regularity assumptions.

Finally, by utilizing the Girsanov transform and adopting a distinct idea from the above works,

- search for some sufficient conditions on the generator such that the corresponding system of quadratic BSDEs admits a (unique) local or global solution for any bounded terminal values rather than some certain terminal values,

Cheridito (2015,Stochastics), Hu and Tang (2016,SPA) and Luo (2020,EJP) respectively established several existence and uniqueness results of local and global solutions for systems of BSDEs with some structured quadratic generators. More specifically,

- Cheridito (2015,Stochastics) investigated system of BSDEs with projectable quadratic generators and subquadratic generators,

- Hu and Tang (2016,SPA) addressed a kind of multi-dimensional BSDEs with diagonally quadratic generators, in which the ith $(i = 1, \cdots, n)$ component g^i of the generator g has a quadratic growth only on the ith row of the matrix z,
- Luo (2020,EJP) considered a class of multi-dimensional BSDEs with triangularly quadratic generators.

We would like to mention that all of these results mentioned above are obtained under the bounded terminal value condition, and

- up to our best knowledge, in existing literature there seems to be no positive general solvability result on the system of quadratic BSDEs with unbounded terminal values.

This talk is the continuation and extension of Hu and Tang (2016,SPA).

Outline

- Let $a \wedge b$ and $a \vee b$ be the minimum and maximum of two real numbers a and b, respectively.
  Set $a^+ := a \vee 0$ and $a^- := -(a \wedge 0)$;
- Denote $\mathbf{1}_A(x) = 1$ when $x \in A$ otherwise 0,
  and $\mathrm{sgn}(x) := \mathbf{1}_{x>0} - \mathbf{1}_{x\leq 0}$;
- The Euclidean norm is always denoted by $|\cdot|$, and $\|\cdot\|_\infty$ denotes the L^∞-norm for one-dimensional or multidimensional random variable.

We define the following four Banach spaces of stochastic processes.

- $\mathcal{S}^p(\mathbb{R}^n)$ for $p \geq 1$ the totality of all $\mathbb{R}^n$-valued continuous adapted processes $(Y_t)_{t\in[0,T]}$ such that

$$\|Y\|_{\mathcal{S}^p} := \left(\mathbb{E}[\sup_{t\in[0,T]} |Y_t|^p]\right)^{1/p} < +\infty.$$

- $\mathcal{S}^\infty(\mathbb{R}^n)$ the totality of all $Y \in \bigcap_{p\geq 1} \mathcal{S}^p(\mathbb{R}^n)$ such that

$$\|Y\|_{\mathcal{S}^\infty} := \operatorname*{ess\,sup}_{(\omega,t)} |Y_t(\omega)| = \left\| \sup_{t\in[0,T]} |Y_t| \right\|_\infty < +\infty.$$

- $\mathcal{H}^p(\mathbb{R}^{n\times d})$ for $p \geq 1$ the totality of all $\mathbb{R}^{n\times d}$-valued $(\mathcal{F}_t)_{t\in[0,T]}$-progressively measurable processes $(Z_t)_{t\in[0,T]}$ such that

$$\|Z\|_{\mathcal{H}^p} := \left\{ \mathbb{E}\left[\left(\int_0^T |Z_s|^2 \mathrm{d}s \right)^{p/2} \right] \right\}^{1/p} < +\infty.$$

- $\mathrm{BMO}(\mathbb{R}^{n\times d})$ the totality of all $Z \in \mathcal{H}^2(\mathbb{R}^{n\times d})$ such that

$$\|Z\|_{\mathrm{BMO}} := \sup_\tau \left\| \mathbb{E}_\tau \left[\int_\tau^T |Z_s|^2 \mathrm{d}s \right] \right\|_\infty^{1/2} < +\infty.$$

The supremum is taken over all $(\mathcal{F}_t)$-stopping times τ with values in $[0,T]$, and $\mathbb{E}_\tau$ denotes the conditional expectation w.r.t. $\mathcal{F}_\tau$.

The spaces $\mathcal{S}^p_{[a,b]}(\mathbb{R}^n)$, $\mathcal{S}^\infty_{[a,b]}(\mathbb{R}^n)$, $\mathcal{H}^p_{[a,b]}(\mathbb{R}^{n\times d})$, and $\mathrm{BMO}_{[a,b]}(\mathbb{R}^{n\times d})$ are identically defined for stochastic processes over the time interval $[a,b]$.

- We note that for $Z \in \mathrm{BMO}(\mathbb{R}^{n\times d})$, the process $\int_0^t Z_s \mathrm{d}B_s, t \in [0,T]$, is an n-dimensional BMO martingale.

For the BMO theory, we refer to the monograph Kazamaki (1994).

- For $i = 1, \cdots, n$, denote by z^i, y^i and g^i resp. the ith row of matrix $z \in \mathbb{R}^{n\times d}$, the ith component of the $y \in \mathbb{R}^n$ and the generator g.

Finally, we write $Y \in \mathcal{E}(\mathbb{R}^n)$ if

$$Y \in \mathbb{R}^n \quad \text{and} \quad \exp(|Y|) \in \bigcap_{p\geq 1} \mathcal{S}^p(\mathbb{R}),$$

and $Z \in \mathcal{M}(\mathbb{R}^{n\times d})$ if

$$Z \in \bigcap_{p\geq 1} \mathcal{H}^p(\mathbb{R}^{n\times d}).$$

Outline

We always fix

- an $(\mathcal{F}_t)$-P.M. scalar-valued non-negative process $(\alpha_t)_{t\in[0,T]}$,
- an increasing continuous function $\phi(\cdot):[0,+\infty)\to[0,+\infty)$ with $\phi(0)=0$,
- several real constants $\beta\geq 0$, $0<\bar{\gamma}\leq\gamma$, $\lambda\geq 0$ and $\delta\in[0,1)$.

We need the following three assumptions.

(H1) For $i=1,\cdots,n$, g^i satisfies that

$\mathrm{d}\mathbb{P}\times\mathrm{d}t-a.e.$, for each $(y,z)\in\mathbb{R}^n\times\mathbb{R}^{n\times d}$,

$$|g^i(\omega,t,y,z)|\leq\alpha_t(\omega)+\phi(|y|)+\frac{\gamma}{2}|z^i|^2+\lambda\sum_{j\neq i}|z^j|^{1+\delta};$$

(H2) For $i = 1, \cdots, n$, g^i satisfies that
$\mathrm{d}\mathbb{P} \times \mathrm{d}t - a.e.$, for each $(y, \bar{y}, z, \bar{z}) \in \mathbb{R}^n \times \mathbb{R}^n \times \mathbb{R}^{n\times d} \times \mathbb{R}^{n\times d}$,

$$\begin{aligned} & |g^i(\omega, t, y, z) - g^i(\omega, t, \bar{y}, \bar{z})| \\ \leq & \, \phi(|y| \vee |\bar{y}|) \Big[(1 + |z| + |\bar{z}|) \left(|y - \bar{y}| + |z^i - \bar{z}^i| \right) \\ & \qquad + \left(1 + |z|^\delta + |\bar{z}|^\delta\right) \sum_{j \neq i} |z^j - \bar{z}^j| \Big]; \end{aligned}$$

(H3) There exists two non-negative constants C_1 and C_2 such that

$$\|\xi\|_\infty \leq C_1 \quad \text{and} \quad \left\| \int_0^T \alpha_t \mathrm{d}t \right\|_\infty \leq C_2.$$

In (H1)-(H2), it creates no essential difference to replace both terms

$\sum_{j\neq i} |z^j|^{1+\delta}$ and $\sum_{j\neq i} |z^j - \bar{z}^j|$ with $|z|^{1+\delta}$ and $|z - \bar{z}|$, respectively.

The underlying way of formulation is more convenient.

Theorem 2.1 (Bounded local solution)

Let assumptions (H1)-(H3) hold. Then, there exist a real $\varepsilon > 0$ (depending only on constants $(n, \gamma, \lambda, \delta, C_1, C_2)$ and function $\phi(\cdot)$) and a closed convex set $\mathcal{B}_\varepsilon$ in the Banach space $\mathcal{S}^\infty_{[T-\varepsilon,T]}(\mathbb{R}^n) \times \mathrm{BMO}_{[T-\varepsilon,T]}(\mathbb{R}^{n\times d})$ such that BSDE (1.1) has a unique local solution (Y, Z) on the time interval $[T-\varepsilon, T]$ with $(Y, Z) \in \mathcal{B}_\varepsilon$.

Note that

$$\mathcal{B}_\varepsilon := \Big\{(U, V) \in \mathcal{S}^\infty(\mathbb{R}^n) \times \mathrm{BMO}(\mathbb{R}^{n\times d}) : \\ \|U\|_{\mathcal{S}^\infty_{[T-\varepsilon,T]}} \le 2K_1 \quad \text{and} \quad \|V\|^2_{\mathrm{BMO}_{[T-\varepsilon,T]}} \le 2K_2\Big\}$$

with the following norm

$$\|(U, V)\|_{\mathcal{B}_\varepsilon} := \sqrt{\|U\|^2_{\mathcal{S}^\infty_{[T-\varepsilon,T]}} + \|V\|^2_{\mathrm{BMO}_{[T-\varepsilon,T]}}}, \quad \forall\ (U, V) \in \mathcal{B}_\varepsilon.$$

We would like to mention that the local solution of Theorem 2.1 is constructed by virtue of uniform a priori estimates on the solution of scalar-valued BSDEs and the fixed-point argument.

We also emphasis that a simpler and more direct idea than that used in the proof of Hu and Tang (2016, SPA; Theorem 2.2) is used to obtain the radius of the centered ball within which the constructed mapping is stable.

Remark 2.2

Assumptions (H1) and (H2) of Theorem 2.1 are more general than those of Theorem 2.2 in Hu and Tang (2016,SPA) in that the former relaxes the growth and continuity of the generator g in y. For example, the following generator g satisfies the former, while not the latter:

$$g^i(\omega, t, y, z) = (|y|^2 + \sin|z^i|)|z| + |z|^{\frac{3}{2}} + |z^i|^2, \quad i = 1, \cdots, n.$$

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The following two assumptions are further required.

(H4) For $i = 1, \cdots, n$, g^i satisfies that

$\mathrm{d}\mathbb{P} \times \mathrm{d}t - a.e.$, for each $(y, z) \in \mathbb{R}^n \times \mathbb{R}^{n\times d}$,

$$\operatorname{sgn}(y^i) g^i(\omega, t, y, z) \leq \alpha_t(\omega) + \beta|y| + \lambda|z|^{1+\delta} + \frac{\gamma}{2}|z^i|^2;$$

(H5) For $i = 1, \cdots, n$, it holds that $\mathrm{d}\mathbb{P} \times \mathrm{d}t - a.e.$,

either for each $(y, z) \in \mathbb{R}^n \times \mathbb{R}^{n\times d}$,

$$g^i(\omega, t, y, z) \geq \frac{\bar{\gamma}}{2}|z^i|^2 - \alpha_t(\omega) - \beta|y| - \lambda|z|^{1+\delta} \tag{4.1}$$

or for each $(y, z) \in \mathbb{R}^n \times \mathbb{R}^{n\times d}$,

$$g^i(\omega, t, y, z) \leq -\frac{\bar{\gamma}}{2}|z^i|^2 + \alpha_t(\omega) + \beta|y| + \lambda|z|^{1+\delta}. \tag{4.2}$$

Remark 2.3

Assumption (H5) holds for the generator g if some components of g satisfy (4.1), and the others satisfy (4.2).

Theorem 2.4 (Bounded global solution I)

Let assumptions (H1)-(H4) be satisfied. If the constant λ in (H4) vanishes, then BSDE (1.1) admits a unique global solution $(Y, Z) \in \mathcal{S}^\infty(\mathbb{R}^n) \times \mathrm{BMO}(\mathbb{R}^{n\times d})$ on $[0, T]$.

Theorem 2.5 (Bounded global solution II)

Let assumptions (H1)-(H5) hold. Then BSDE (1.1) admits a unique global solution $(Y, Z) \in \mathcal{S}^\infty(\mathbb{R}^n) \times \mathrm{BMO}(\mathbb{R}^{n\times d})$ on $[0, T]$.

Remark 2.6

Assumption (H4) is some kind of one-sided linear growth condition of g in y, and (H5) can be regarded as some kind of strictly quadratic condition of g^i w.r.t. z^i. A generator g satisfying (H1)-(H5) can still have a general growth in y. For example, the following g satisfies all these assumptions:

$$g^i(\omega, t, y, z) = (\mathrm{e}^{-y^i} + \cos|z^i|)|z| - |z|^{\frac{4}{3}} + (-1)^i|z^i|^2, \quad i = 1, \cdots, n.$$

This g does not satisfy the assumptions in Hu and Tang (2016,SPA).

Lemma (Uniform a priori estimate)

Let (H3) hold. Assume that for some $h \in (0, T]$, the BSDE (1.1) has a solution $(Y, Z) \in \mathcal{S}^\infty_{[T-h,T]} \times \mathrm{BMO}_{[T-h,T]}$ on $[T-h, T]$. Then

(i) If the generator g satisfies (H4) with $\lambda = 0$, then

$$\|Y\|_{\mathcal{S}^\infty_{[T-h,T]}} \leq n(C_1 + C_2)\exp(n\beta T).$$

(ii) If the generator g satisfies (H4) and (H5), then

$$\|Y\|_{\mathcal{S}^\infty_{[T-h,T]}} \leq 2n(C_1 + C_3)\exp(2n\beta T),$$

where C_3 is a positive constant depending only on $(n, \beta, \gamma, \bar{\gamma}, \lambda, \delta, T, C_2)$.

Proof of Theorems 2.4 and 2.5. Using Theorem 2.1 and this Lemma, we can follow the proof of Theorem 4.1 in Cheridito (2015,Stochastics) to derive our Theorems 2.4 and 2.5. All the details are omitted here.

Outline

We further introduce the following four assumptions.

(B1) For $i = 1, \cdots, n$, $g^i(\omega, t, y, z)$ varies with (ω, t, y) and the ith row z^i of the matrix $z \in \mathbb{R}^{n\times d}$ only, and grows linearly in y and quadratically in z^i, i.e., $\mathrm{d}\mathbb{P} \times \mathrm{d}t - a.e.$,

$$|g^i(\omega, t, y, z^i)| \le \alpha_t(\omega) + \beta|y| + \frac{\gamma}{2}|z^i|^2 \ \text{ for each } (y, z) \in \mathbb{R}^n \times \mathbb{R}^{n\times d};$$

(B2) g is uniformly Lipschitz continuous in y, i.e., $\mathrm{d}\mathbb{P} \times \mathrm{d}t - a.e.$,

$$|g(\omega, t, y, z) - g(\omega, t, \bar{y}, z)| \le \beta|y - \bar{y}| \ \text{ for each } (y, \bar{y}, z) \in (\mathbb{R}^n)^2 \times \mathbb{R}^{1\times d};$$

(B3) $\mathrm{d}\mathbb{P} \times \mathrm{d}t - a.e.$, for each $i = 1, \cdots, n$ and $y \in \mathbb{R}^n$, $g^i(\omega, t, y, \cdot)$ is either convex or concave;

(B4) The terminal value ξ is of exponential moments of arbitrary order as well as $\int_0^T \alpha_t \mathrm{d}t$. That is, we have for each $p \ge 1$,

$$\mathbb{E}\left[\exp\left\{p\left(|\xi| + \int_0^T \alpha_t \mathrm{d}t\right)\right\}\right] < +\infty.$$

Remark 2.7

Assumption (B3) holds for the generator g if some components of g are convex in z, and the others are concave in z.

Theorem 2.8 (Unbounded global solution)

Let assumptions (B1)-(B4) be in force. Then BSDE (1.1) admits a unique global solution $(Y, Z) \in \mathcal{E}(\mathbb{R}^n) \times \mathcal{M}(\mathbb{R}^{n\times d})$ on $[0, T]$.

Remark 2.9

In Theorem 2.8, the martingale part of the first unknown process Y goes beyond the space of BMO martingales, and some delicate and technical computations are developed in its proof, in which

- a priori estimates on one-dimensional quadratic BSDEs,
- the θ-method for convex functions,
- Doob's maximal inequality for martingales

play a crucial role.

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Thanks for your kind attention!